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**Autor:** Eliahou, Shalom / Kervaire, Michel  
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Finally, in Section 4 and 5, we give several examples of the use of the Multiplier Theorem.

## 0. PRELIMINARIES

In this section, we establish the simple relationship between periodic and aperiodic correlation coefficients, and show that every Barker sequence of length greater than 2 is also a *periodic* Barker sequence.

**LEMMA.** *Let  $A = (a_1, \dots, a_l)$  be a binary sequence. Then*

$$\gamma_j(A) = c_j(A) + c_{l-j}(A)$$

for all  $j = 1, \dots, l - 1$ .

*Proof.* We have

$$\begin{aligned} \gamma_j(A) &= \sum_{i=1}^l a_i a_{i+j} = \sum_{i=1}^{l-j} a_i a_{i+j} + \sum_{i=l-j+1}^l a_i a_{i+j} \\ &= c_j(A) + \sum_{i=l-j+1}^l a_{i+j-l} a_i = c_j(A) + c_{l-j}(A), \end{aligned}$$

as claimed.  $\square$

For the next result, we will use, as other papers on binary sequences do, the simple observation that

$$ab \equiv a + b - 1 \pmod{4}$$

for all  $a, b \in \{+1, -1\}$ .

**PROPOSITION 2.** *Let  $A = (a_1, \dots, a_l)$  be a Barker sequence, with  $l \geq 3$ . Then  $A$  is also a periodic Barker sequence.*

*Proof.* We have to prove that  $\gamma_j = \gamma_j(A)$  is independent of  $j = 1, \dots, l - 1$ , and equal to 0 or  $\pm 1$ . First of all, we have (with  $c_j = c_j(A)$ )

$$(1) \quad c_j = \begin{cases} 0 & \text{if } l - j \text{ is even} \\ \pm 1 & \text{if } l - j \text{ is odd} \end{cases}$$

for all  $j = 1, \dots, l - 1$ .

This follows from the obvious congruence  $c_j \equiv l - j \pmod{2}$ , and the fact that  $c_j \in \{-1, 0, +1\}$ , for all  $j = 1, \dots, l - 1$ .

Now, applying the relation  $ab \equiv a + b - 1 \pmod{4}$  for any  $a, b = \pm 1$ , we have

$$(2) \quad c_j = \sum_{i=1}^{l-j} a_i a_{i+j} \equiv \sum_{i=1}^{l-j} (a_i + a_{i+j}) - (l-j) \pmod{4}$$

for  $j = 1, \dots, l - 1$ .

Comparing the above congruences for two successive values of  $j$ , we obtain

$$(3) \quad c_j - c_{j+1} \equiv a_{l-j} + a_{j+1} - 1 \pmod{4},$$

for  $j = 1, \dots, l - 2$ .

Changing  $j$  to  $l - j - 1$  leaves the right-hand-side unchanged. Therefore, we have

$$(4) \quad c_j - c_{j+1} \equiv c_{l-j-1} - c_{l-j} \pmod{4},$$

for  $j = 1, \dots, l - 2$ . Since  $|c_j - c_{j+1}| \leq 1$  for all  $j$  by (1), we have in fact an equality:

$$c_j - c_{j+1} = c_{l-j-1} - c_{l-j}$$

for  $j = 1, \dots, l - 2$ . Using Lemma 1, it follows that

$$\gamma_j = \gamma_{j+1}$$

for all  $j = 1, \dots, l - 2$ , and thus  $\gamma_j$  is independent of  $j$ , as claimed.

Now  $|\gamma_j| = |c_j + c_{l-j}| \leq 2$ , and equality can occur only if  $c_j = c_{l-j} = \pm 1$ , which by (1) implies in particular that  $j$  must be odd. But this is impossible, because  $\gamma_j$  is independent of  $j$ . Therefore  $|\gamma_j| \leq 1$ , as claimed.  $\square$

## 1. DIFFERENCE SETS

In this section, we show that the notion of a binary sequence with constant periodic correlations is equivalent to that of a difference set on a cyclic group. We then recall basic results concerning these difference sets.

*Definition.* A *difference set*  $D$  on a group  $G$  is a subset  $D \subset G$  such that the cardinality of the intersection

$$D \cap g \cdot D$$

is independent of  $g$  for  $g \in G \setminus \{e\}$ . Here,  $gD = \{gx \mid x \in D\}$  is the translate of  $D$  by the element  $g \in G$ , and  $e$  is the neutral element of  $G$ .