Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 38 (1992)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: BARKER SEQUENCES AND DIFFERENCE SETS
Autor: Eliahou, Shalom / Kervaire, Michel
Kapitel: Introduction
DOI: https://doi.org/10.5169/seals-59496

Nutzungsbedingungen
Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Mehr erfahren

Conditions d'utilisation
L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. En savoir plus

Terms of use
The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. Find out more

Download PDF: 23.01.2026

ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch

L'Enseignement Mathématique, t. 38 (1992), p. 345-382

BARKER SEQUENCES AND DIFFERENCE SETS

by Shalom ELIAHOU and Michel KERVAIRE

INTRODUCTION

This paper deals with binary sequences $A = (a_1, \ldots, a_l)$, i.e. $a_i = \pm 1$ for $i = 1, \ldots, l$, and classical coefficients associated with them, the aperiodic and periodic correlation coefficients. The *aperiodic correlation coefficients* of A are defined as

$$c_j(A) = \sum_{i=1}^{l-j} a_i a_{i+j}\,, \qquad \text{for} \quad j = 1, \ldots, l-1\,,$$

and the *periodic correlation coefficients* of A as

$$\gamma_j(A) = \sum_{i=1}^{l} a_i a_{i+j}, \qquad \text{for} \quad j = 1, \ldots, l-1\,,$$

where the indices are read modulo l, i.e. $a_r = a_{r-l}$ if $r \geqslant l+1$.

It is well-known that $\gamma_j = c_j + c_{l-j}$ for $j = 1, \ldots, l-1$.

There are many interesting and difficult problems concerned with the existence of binary sequences whose correlation coefficients (or *correlations,* for short) are subject to various conditions. We will examine here three classical situations.

(1) One may require the *periodic* correlations γ_j to be constant, i.e.

$$\gamma_1 = \gamma_2 = \ldots = \gamma_{l-1} = \gamma\,.$$

We will see below that binary sequences satisfying this condition are equivalent to the classical notion of cyclic difference sets.

(2) In addition to the condition above, one may furthermore impose the constant γ to be small, i.e. $\gamma = 0$, or $\gamma = 1$, or $\gamma = -1$. We will call such

The authors gratefully acknowledge partial support from the Fonds National Suisse de la Recherche Scientifique during the preparation of this paper.

sequences *periodic Barker sequences*. Periodic Barker sequences with $\gamma = 0$ are equivalent to circulant Hadamard matrices. We will not follow the link with Hadamard matrices any further here.

(3) Without any condition on the periodic correlations, one may require the *aperiodic* correlations to be small, i.e.

$$c_j \in \{0, 1, -1\} \quad \text{for} \quad j = 1, \ldots, l-1 .$$

Such sequences are known as *Barker sequences*. They were invented by Barker [Bar] in connection with radar theory. Note that we cannot impose the c_j to be constant, since $c_j \equiv l - j \bmod 2$.

Barker sequences of *odd length* have been classified in 1961 by Storer and Turyn [ST]. Their lengths are bounded by 13. In the *even length* case, a longstanding conjecture states that the only such sequences are of length 2 or 4. It is known since Turyn [T2] that if the length of a Barker sequence is even and greater than 4, then it must be at least 12 100. We will show in Section 3 that this lower bound can be improved to 1 898 884, thanks to a recent result on Golay pairs and Barker sequences [EKS], and an observation in [JL].

Here is a summary of the content of this paper. In Section 0, we prove that Barker sequences of length greater than 2 are in fact *periodic* Barker sequences (i.e., (3) $\Rightarrow$ (2)), an elementary and well known fact. It is sometimes asserted in the literature that the converse holds as well. This is not true, and clarifying the situation was one of our motivations to write this survey. Another motivation was our exploration of the existence question of periodic Barker sequences for an extensive range of possible lengths. This work is summarized in Tables I and II at the end of the paper.

In Section 1, we show that binary sequences with constant periodic correlations (condition (1)) are equivalent to cyclic difference sets. We then recall the main results concerning these difference sets.

Section 2 deals with condition (2), that is, *periodic* Barker sequences. We examine the cases $\gamma = 0, 1$ and -1 separately. In the case $\gamma = 0$, it is widely believed that the only possible length is $l = 4$. We recall a theorem of Turyn stating that l must be of the form $l = 4N^2$, where N is an odd integer. Further results of Turyn imply that N must necessarily be greater than or equal to 55. In the case $\gamma = 1$, there is only one known example. The case $\gamma = -1$, in contrast, provides many interesting classical examples. In that case we make explicit the complete classification of $(4n - 1, 2n - 1, n - 1)$ cyclic difference sets up to $n = 100$. (See Sections 4 and 5, and Tables I and II.)

In Section 3, we show that there exists no aperiodic Barker sequence of length divisible by $2p$, when p is a prime number congruent to 3 mod 4.

Finally, in Section 4 and 5, we give several examples of the use of the Multiplier Theorem.

0. Preliminaries

In this section, we establish the simple relationship between periodic and aperiodic correlation coefficients, and show that every Barker sequence of length greater than 2 is also a *periodic* Barker sequence.

Lemma. *Let* $A = (a_1, \ldots, a_l)$ *be a binary sequence. Then*

$$\gamma_j(A) = c_j(A) + c_{l-j}(A)$$

for all $j = 1, \ldots, l-1$.

Proof. We have

$$\gamma_j(A) = \sum_{i=1}^{l} a_i a_{i+j} = \sum_{i=1}^{l-j} a_i a_{i+j} + \sum_{i=l-j+1}^{l} a_i a_{i+j}$$

$$= c_j(A) + \sum_{i=l-j+1}^{l} a_{i+j-l} a_i = c_j(A) + c_{l-j}(A)\,,$$

as claimed. □

For the next result, we will use, as other papers on binary sequences do, the simple observation that

$$ab \equiv a + b - 1 \bmod 4$$

for all $a, b \in \{+1, -1\}$.

Proposition 2. *Let* $A = (a_1, \ldots, a_l)$ *be a Barker sequence, with* $l \geqslant 3$. *Then* A *is also a periodic Barker sequence.*

Proof. We have to prove that $\gamma_j = \gamma_j(A)$ is independent of $j = 1, \ldots, l-1$, and equal to 0 or ± 1. First of all, we have (with $c_j = c_j(A)$)

$$c_j = \begin{cases} 0 & \text{if} \quad l-j \text{ is even} \\ \pm 1 & \text{if} \quad l-j \text{ is odd} \end{cases} \tag{1}$$

for all $j = 1, \ldots, l-1$.