

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 37 (1991)  
**Heft:** 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** ON THE FACTORIZATION OF  $X^n - BX - A$   
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**DOI:** <https://doi.org/10.5169/seals-58737>

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## ON THE FACTORIZATION OF $X^n - BX - A$

by P. RIBENBOIM

1. Rabinowitz [Ra] proved that the only integers  $A$ , for which  $X^5 - X - A$  is a product of an irreducible quadratic and an irreducible cubic polynomial with coefficients in  $\mathbf{Z}$ , are  $A = \pm 15$ ,  $\pm 22440$ , and  $\pm 2759640$ . The factorizations are

$$X^5 - X \pm 15 = (X^2 \pm X + 3) (X^3 \mp X^2 - 2X \pm 5) ,$$

$$X^5 - X \pm 22440 = (X^2 \mp 12X + 55) (X^3 \pm 12X^2 + 89X \pm 408) ,$$

$$X^5 - X \pm 2759640 = (X^2 \pm 12X + 377) (X^3 \mp 12X^2 - 233X \pm 7320) .$$

Similarly

$$X^5 + X \pm 1 = (X^2 \pm X + 1) (X^3 \mp X^2 \pm 1) ,$$

$$X^5 + X \pm 6 = (X^2 \pm X + 2) (X^3 \mp X^2 - X \pm 3)$$

are the only similar decompositions for polynomials  $X^5 + X - A$ .

This rather interesting result requires, in last analysis, the fact that 1, 144 are the only non-zero Fibonacci numbers which are squares.

We shall extend this result for the polynomials  $X^n - BX - A$ , where  $A$  is a given non-zero integer and  $n \geq 5$ , and also for the polynomials of the same type, where  $B$  is a given non-zero integer, and  $n > 5$ .

The proof is elementary, except in one of the cases, where Thue's theorem (see [Th]) is required. Due to Baker's work (see [Sh-Ti], page 99), an explicit bound for the solutions of Thue's equation is now known, making our result also effective.

I am grateful to J. Top for discussions about this paper.

The proof is elementary, except for the use of Thue's theorem and a theorem of Pethö, Shorey & Stewart concerning the squares in recurring sequences.

2. For the convenience of the reader, we recall all needed facts.

Thue's theorem [Th] states:

Let  $G \in \mathbf{Z}[X]$  be a polynomial with at least three distinct roots, let  $g(X, Y)$  be the associated homogeneous polynomial.

For every integer  $m$ , there exist at most finitely many pairs of integers  $(x, y)$  such that  $g(x, y) = m$ . Due to the work of Baker (see [Sh-Ti], page 99), an explicit bound for the solutions  $x, y$  of Thue's equation is now known.

We shall encounter also the diophantine equation  $X^2 - 5Y^4 = 4B$ . This will lead to the study of  $X^2 - 5Y^2 = 4B$  and to the determination of its solutions  $(x, y)$  such that  $y$  is a square.

The equation  $X^2 - 5Y^2 = 4B$  has been studied by Stolt [St] and we gather here the results to be needed.

Let  $S = \left\{ \alpha = \frac{a + b\sqrt{5}}{2} \mid a, b \in \mathbf{Z}, a^2 - 5b^2 = 4B \right\}$ . If  $S \neq \emptyset$  then  $B \neq 0$ ,

and if  $\frac{a + b\sqrt{5}}{2} \in S$  then  $a \equiv b \pmod{2}$ . Also  $b = 0$  exactly when  $B$  is a

square, and if  $\frac{a + b\sqrt{5}}{2}, \frac{a' + b'\sqrt{5}}{2} \in S$ , then  $a = \pm a'$ .

We recall that the units of the number field  $\mathbf{Q}(\sqrt{5})$  are  $\pm \omega^n$  (for  $n \in \mathbf{Z}$ ), where  $\omega = \frac{1 + \sqrt{5}}{2}$ ; since  $\omega$  has norm equal to  $-1$ , then the units of norm 1

are  $\pm \zeta^n$  (for  $n \in \mathbf{Z}$ ) where  $\zeta = \omega^2 = \frac{3 + \sqrt{5}}{2}$ .

We say that  $\alpha, \alpha' \in S$  are equivalent when  $\frac{\alpha}{\alpha'} = \pm \zeta^n$  (where  $n \in \mathbf{Z}$ ). We

say that  $\alpha = \frac{a + b\sqrt{5}}{2} \in S$  is fundamental when  $0 \leq b$  and if  $\alpha \sim \alpha'$

$= \frac{a' + b'\sqrt{5}}{2}$  then  $b \leq |b'|$ . Thus each equivalence class contains one, and

at most four fundamental elements.

Now we show that there are only finitely many equivalence classes in  $S$ ; more explicitly, the number of equivalence classes is at most equal to  $\sqrt{|B|}$ .

It suffices to show that if  $\alpha = \frac{a + b\sqrt{5}}{2}$  is a fundamental element, then

$b \leq \sqrt{|B|}$ . Indeed  $\frac{a + b\sqrt{5}}{2} + \frac{3 + \sqrt{5}}{2} = \frac{1}{2} \left( \frac{3a + 5b}{2} + \frac{3b + a}{2} \sqrt{5} \right)$  and

$\frac{a + b\sqrt{5}}{2} + \frac{3 - \sqrt{5}}{2} = \frac{1}{2} \left( \frac{3a - 5b}{2} + \frac{3b - a}{2} \sqrt{5} \right)$ . Since  $\frac{a + b\sqrt{5}}{2}$  is funda-

mental, then  $b \leq \left| \frac{3b + a}{2} \right|$ ,  $b \leq \left| \frac{3b - a}{2} \right|$ .

If  $0 \leq \frac{3b+a}{2}$  then  $-a \leq b$  hence  $5b^2 + 4B = a^2 \leq b^2$  so  $b^2 \leq -B = |B|$  hence  $b \leq \sqrt{|B|}$ . If  $\frac{3b+a}{2} \leq 0$  then  $5b \leq -a$  and  $25b^2 \leq a^2 = 5b^2 + 4B$  so  $b \leq \sqrt{\frac{B}{5}} \leq \sqrt{|B|}$ . The other cases give the same bound for  $b$ .

Now, we consider the equivalence class of the fundamental element  $\frac{a+b\sqrt{5}}{2}$ . Define the integers  $x_n, y_n$  (for every  $n \in \mathbf{Z}$ ) by the relation

$$\frac{a+b\sqrt{5}}{2} \left(\frac{3+\sqrt{5}}{2}\right)^n = \frac{x_n+y_n\sqrt{5}}{2}.$$

Since  $\frac{a-b\sqrt{5}}{2} \left(\frac{3-\sqrt{5}}{2}\right)^n = \frac{x_n-y_n\sqrt{5}}{2}$  then  $\frac{a^2-5b^2}{4} = \frac{x_n^2-5y_n^2}{4} = B$ .

And from what precedes, if  $x^2 - 5y^2 = 4B$  there exists a fundamental element  $\frac{a+b\sqrt{5}}{2}$  and  $n \in \mathbf{Z}$  such that  $x = \pm x_n, y = \pm y_n$ .

We may describe the sequences  $(x_n)_{n \in \mathbf{Z}}$  and  $(y_n)_{n \in \mathbf{Z}}$  by linear recurrences of order 2.

Let  $U_0(3, 1) = 0, U_1(3, 1) = 1$  and for  $n \geq 2, U_n(3, 1) = 3U_{n-1}(3, 1) - U_{n-2}(3, 1)$ , while for  $n < 0, U_n(3, 1) = -U_{-n}(3, 1)$ .

Similarly, let  $V_0(3, 1) = 2, V_1(3, 1) = 3$ , for  $n \geq 2, V_n(3, 1) = 3V_{n-1}(3, 1) - V_{n-2}(3, 1)$ , while for  $n < 0, V_n(3, 1) = V_{-n}(3, 1)$ .

With this notation, we verify by a simple induction, that

$$\begin{cases} 2x_n = V_n(3, 1)a + 5U_n(3, 1)b, \\ 2y_n = U_n(3, 1)a + V_n(3, 1)b. \end{cases}$$

We are interested in finding an effective bound for  $n \geq 1$  such that  $y_n$  is a square.

But 
$$U_n(3, 1) = \frac{\zeta^n - \zeta^{-n}}{\sqrt{5}}, \quad V_n(3, 1) = \zeta^n + \zeta^{-n}$$

hence 
$$2y_n = \frac{a}{\sqrt{5}} (\zeta^n - \zeta^{-n}) + b(\zeta^n + \zeta^{-n})$$

$$= \left(\frac{a}{\sqrt{5}} + b\right) \zeta^n - \left(\frac{a}{\sqrt{5}} - b\right) \zeta^{-n}.$$

By the theorem of Pethö [Pe] and Shorey & Stewart [Sh-St] (see also [Sh-Ti], theorem 9.6) there exists an effective constant  $C(a, b) > 0$  such that if  $\left(\frac{a}{\sqrt{5}} + b\right) \zeta^n - \left(\frac{a}{\sqrt{5}} - b\right) \zeta^{-n} = 2\Box$  (twice a square), then  $n < C(a, b)$ .

Letting  $C = \max\{C(a, b) \mid \frac{a + b\sqrt{5}}{2} \text{ is a fundamental element of } S\}$ , then if  $x^2 - 5y^4 = 4B$  it follows that  $|y|$  is effectively bounded, since  $y = y_n = 2\Box$  (for some fundamental element  $\frac{a + b\sqrt{5}}{2} \in S$ ).

3. Here is our proposition:

PROPOSITION. *Let  $n \geq 5$ .*

1) *For every non-zero integer  $A$ , there exists an effectively determined integer  $\beta > 0$ , such that if  $X^n - BX - A \in \mathbf{Z}[X]$  has a quadratic factor in  $\mathbf{Z}[X]$  which is monic, then  $|B| < \beta$ .*

2) *For every non-zero integer  $B$ , there exists an effectively determined integer  $\alpha > 0$ , such that if  $X^n - BX - A \in \mathbf{Z}[X]$  has a quadratic factor in  $\mathbf{Z}[X]$  which is monic, then  $|A| < \alpha$ .*

*Proof.* Write

$$X^n - BX - A = (X^2 - bX - a)(X^{n-2} + c_{n-3}X^{n-3} + \cdots + c_1X + c_0),$$

where  $a, b, c_i \in \mathbf{Z}$ .

Then

$$A = ac_0$$

$$B = ac_1 + bc_0$$

$$0 = -ac_2 - bc_1 + c_0$$

$$0 = -ac_3 - bc_2 + c_1$$

.....

$$0 = -ac_{n-3} - bc_{n-4} + c_{n-5}$$

$$0 = -a - bc_{n-3} + c_{n-4}$$

$$0 = -b + c_{n-3}.$$

From these relations, we obtain successively

$$c_{n-3} = b$$

$$c_{n-4} = a + bc_{n-3} = a + b^2$$

$$c_{n-5} = ac_{n-3} + bc_{n-4} = 2ab + b^3 .$$

$$c_{n-6} = ac_{n-4} + bc_{n-5} = a^2 + 3ab^2 + b^4$$

.....

$$c_1 = ac_3 + bc_2,$$

$$c_0 = ac_2 + bc_1,$$

$$B = ac_1 + bc_0,$$

$$A = ac_0.$$

In order to determine explicitly  $c_i$  in terms of  $a, b$ , consider the following linear recurring sequence of polynomials:

$$F_0(X) = 1, F_1(X) = 1, \text{ and for every } i \geq 2, F_i(X) = F_{i-1}(X) + XF_{i-2}(X).$$

By induction, it may be seen that  $F_i(X)$  has degree  $j = \left\lfloor \frac{i}{2} \right\rfloor$ . Moreover, if  $i$  is even then

$$F_i(X) = X^j + \binom{j+1}{j-1} X^{j-1} + \binom{j+2}{j-2} X^{j-2} + \dots + \binom{j+k}{j-k} X^{j-k}$$

$$+ \dots + \binom{2j-1}{1} X + 1$$

and if  $i$  is odd then

$$F_i(X) = \binom{j+1}{j} X^j + \binom{j+2}{j-1} X^{j-1} + \dots + \binom{j+k}{j-k+1} X^{j-k+1}$$

$$+ \dots + \binom{2j}{1} X + 1 .$$

Note that  $F_i(0) = 1, F_i(1) > 0$  for every  $i \geq 0$ . Also, if  $r \in \mathbf{Z}$  and  $F_i(r) = 0$  then  $r = -1$ .

Let  $f_i(X, Y) = Y^j F_i\left(\frac{X}{Y}\right)$  so  $f_i(X, Y)$  is a homogeneous polynomial of degree  $j$ . As easily seen,

$$f_i(X, Y) = \begin{cases} f_{i-1}(X, Y) + Xf_{i-2}(X, Y) & \text{when } i \text{ is odd} \\ Yf_{i-1}(X, Y) + Xf_{i-2}(X, Y) & \text{when } i \text{ is even} . \end{cases}$$

Hence

$$c_{n-3} = b = bf_1(a, b^2)$$

$$c_{n-4} = a + b^2 = f_2(a, b^2)$$

$$c_{n-5} = b(2a + b^2) = bf_3(a, b^2)$$

$$c_{n-6} = a^2 + 3ab^2 + b^4 = f_4(a, b^2)$$

.....

$$c_1 = ac_3 + bc_2 = \begin{cases} f_{n-3}(a, b^2) & \text{when } n \text{ is odd} \\ bf_{n-3}(a, b^2) & \text{when } n \text{ is even,} \end{cases}$$

$$c_0 = ac_2 + bc_1 = \begin{cases} bf_{n-2}(a, b^2) & \text{when } n \text{ is odd} \\ f_{n-2}(a, b^2) & \text{when } n \text{ is even,} \end{cases}$$

$$B = ac_1 + bc_0 = \begin{cases} af_{n-3}(a, b^2) + b^2f_{n-2}(a, b^2) = f_{n-1}(a, b^2) & \text{when } n \text{ is odd} \\ abf_{n-3}(a, b^2) + bf_{n-2}(a, b^2) = bf_{n-1}(a, b^2) & \text{when } n \text{ is even,} \end{cases}$$

$$A = ac_0 = \begin{cases} abf_{n-2}(a, b^2) & \text{when } n \text{ is odd} \\ af_{n-2}(a, b^2) & \text{when } n \text{ is even.} \end{cases}$$

First let  $n$  be even. Given  $A$ ,  $a$  belongs to the finite set of integers dividing  $A$ ; thus  $b$  belongs to the finite set of integers which are solutions of any one of the equations  $af_{n-2}(a, Y^2) = A$ . Therefore  $B$ , which is expressed in terms of  $a, b$ , belongs to a finite set.

Given  $B$ ,  $b$  belongs to the finite set of integers dividing  $B$ ; thus  $a$  belongs to the finite set of integers which are solutions of any one of the equations  $bf_{n-1}(X, b^2) = B$ . Therefore  $A$ , which is expressed in terms of  $a, b$ , belongs to a finite set.

Now, let  $n$  be odd. Given  $A$ , both  $a$  and  $b$  belong to the finite set of divisors of  $A$ . Therefore  $B$ , which is expressible in terms of  $a, b$ , belongs to a finite set too.

Finally, we treat the more interesting case, where  $n$  is odd,  $n \geq 5$  and  $B$  is given. First let  $n \geq 7$ . Now  $F_{n-1}(X)$ , has degree  $M = \frac{n-1}{2} \geq 3$ .

We consider the following cases.

- 1)  $F_{n-1}(X)$  has an irreducible factor in  $\mathbf{Z}[X]$ , of degree at least 3.
- 2)  $F_{n-1}(X)$  has at least two distinct irreducible factors in  $\mathbf{Z}[X]$ , each of degree 2.
- 3)  $F_{n-1}(X)$  has an irreducible factor of degree 2 and a linear factor in  $\mathbf{Z}[X]$ .

- 4)  $F_{n-1}(X)$  has at least three distinct linear factors in  $\mathbf{Z}[X]$ .
- 5)  $F_{n-1}(X)$  is a power of an irreducible polynomial of degree 2 in  $\mathbf{Z}[X]$ .
- 6)  $F_{n-1}(X)$  has exactly two distinct linear factors in  $\mathbf{Z}[X]$ .
- 7)  $F_{n-1}(X)$  is a power of a linear factor in  $\mathbf{Z}[X]$ .

In cases (1), (2), (3), (4),  $F_{n-1}(X)$  has a factor  $G(X) \in \mathbf{Z}[X]$  with at least three distinct roots. Let  $g(X, Y)$  be the homogeneous polynomial associated to  $G(X)$ .

Then  $a, b$  belong to the set  $I$  of integers such that  $g(a, b^2)$  is a divisor of  $B$ . By Thue's theorem (in its effective version), there is an effective bound for the possible integers  $a, b$ , thus  $a, b$  belong to a finite set, and therefore  $A$  belongs to finite set too.

In case (5),  $F_{n-1}(X) = (X^2 + rX + s)^k$ . Comparing degrees,  $\frac{n-1}{2} = m = 2k$  and comparing the constant terms,  $1 = s^k$ , hence  $s = \pm 1$ . Comparing the coefficients of  $X^{n-1}$ , we have:  $\binom{m+1}{m-1} = kr$ , hence  $\frac{(m+1)m}{2} = \frac{mr}{2}$ , so  $r = m + 1$ .

Comparing the coefficients of  $X^{m-2}$ , we have:  $\binom{m+2}{m-2} = ks + \binom{k}{2} r^2$ , hence

$$\frac{(m+2)(m+1)m(m-1)}{24} = \pm \frac{m}{2} + \frac{m(m-2)(m+1)^2}{8}$$

and this gives

$$m^3 - m^2 - 4m + 4 = 0,$$

respectively

$$m^3 - m^2 - 4m - 8 = 0.$$

The first equation has only solutions  $m = 1, m = 2$  in positive integers — but this has been excluded.

The second equation has no solution in positive integers. Therefore, the case (5) cannot happen.

In case (6),  $F_{n-1}(X) = (X+r)^k(X+s)^h$  with  $r, s \in \mathbf{Z}, r \neq s$ . Then  $m = k + h$ . Comparing the constant term, we have  $1 = r^k s^h$ , so  $r, s = \pm 1$ , and therefore say,  $r = 1, s = -1$ .

Comparing the coefficients of  $X^{m-1}$ ,  $\binom{m+1}{m-1} = kr + hs$ , hence  $\frac{m(m+1)}{2} = k - h$ . But  $k - h < k + h = m < \frac{(m+1)m}{2}$ , so this case is impossible.

Finally, in case (7),  $F_{n-1}(X) = (X+r)^k$ , with  $r \in \mathbf{Z}$ . Comparing degrees, constant terms and coefficients of  $X^{m-1}$ , we have  $m = k$ ,  $1 = r^k$ , so  $r = \pm 1$ , and  $\binom{m+1}{m-1} = kr$ , so  $\frac{(m+1)m}{2} = \pm m$ ; this gives  $m = 1$ , which is excluded.

It remains to treat the case  $n = 5$ . Then  $F_4(X) = X^2 + 3X + 1$ , so  $f_4(X, Y) = X^2 + 3XY + Y^2$ . Given  $B$ , we consider the set  $E$  of all pairs of integers  $(a, b)$  such that  $f_4(a, b^2) = B$ , that is  $a^2 + 3ab^2 + b^4 = B$ ; this may be rewritten as  $\left(a + \frac{3}{2}b\right)^2 - \frac{5}{4}b^2 = B$ , hence  $x^2 - 5y^2 = 4B$ , where,  $x = 2a + 3b, y = b^2$ .

As it was indicated in §2, there is an explicitly computable constant  $C > 0$ , such that if  $(x, y)$  satisfies the above relations, then  $y < C$ , this yields explicit bounds for  $b, x$  and therefore also for  $a$ .

This concludes the proof.

### Remarks.

1) An effective bound for the size of solutions of Thue's equation is indicated, for example, in [Sh-Ti], page 99. It is far too large for any practical purpose. It should however be noted that what is required is to determine the solutions in integers  $x, y$  of the equations  $g(X, Y) = m$  (for every divisor  $m$  of  $B$ ), such that  $y$  is a square.

If  $n = 5$  and  $B = \pm 1$ , the calculations lead to  $\pm 1 = B = b^4 + 3ab^2 + a^2$  and  $A = ab(2a + b^2)$ , hence

$$\left(a + \frac{3}{2}b^2\right)^2 - \frac{5b^4}{4} = \pm 1,$$

so

$$(2a + 3b^2)^2 - 5b^4 = \pm 4.$$

The solutions of  $X^2 - 5Y^2 = \pm 4$  are known to be  $x = L_{2n}, y = F_{2n}$  (for the + sign),  $x = L_{2n+1}, y = F_{2n+1}$  (for the - sign), for every  $n \geq 0$ ; here

$F_k, L_k (k \geq 0)$  are respectively the Fibonacci and the Lucas numbers. So  $b^2$  is a Fibonacci number. As it is well known,  $b^2 = 1 = F_1 = F_2$  or  $b^2 = 144 = F_{12}$ , and this leads eventually to the decompositions indicated by Rabinowitz.

2) Let  $n \geq 5$  and  $E = \{(A, B) \in \mathbf{Z} \times \mathbf{Z} \mid X^n - BX - A \text{ has a factor of degree 2 in } \mathbf{Z}[X]\}$ ; for each  $A, B \in \mathbf{Z}$ , let  $E'_A = \{B \in \mathbf{Z} \mid (A, B) \in E\}$ ,  $E''_B = \{A \in \mathbf{Z} \mid (A, B) \in E\}$ .

It is easy to see that  $E$  is an infinite set. Indeed, if  $a, b \in \mathbf{Z}$ , let

$$X^n = q(X^2 - bX - a) + BX + A, \quad \text{where}$$

$q \in \mathbf{Z}[X]$ , then  $X^2 + bX + a$  divides  $X^n - BX - A$ . Since each polynomial  $X^n - BX - A$  has at most finitely many factors of second degree, then the set  $E$  is infinite.

The propositions proved in the paper state that each set  $E'_A, E''_B$  (for  $n \geq 5$ ) is finite, and also its members may be found effectively. However it is not ruled out that  $E'_A$  or  $E''_B$  be empty for values of  $A$  or  $B$ .

It is feasible to determine congruence conditions on  $A$ , resp.  $B$  which must be satisfied if  $E'_A \neq \emptyset$ , respectively  $E''_B \neq \emptyset$ .

Calculations made at my request by Y. Gérard, indicated that if  $n = 5$  and  $E''_B \neq \emptyset$  then  $B \equiv 0, \pm 1 \pmod{5}$ . Gérard has also noted that if  $B \equiv \pm 1 \pmod{5}$  and there exists a prime  $p$  dividing  $B$  and  $p \equiv \pm 2 \pmod{5}$  then  $E''_B = \emptyset$ .

For  $B = -11, -19, -29, -31$ , the following factorizations hold

$$X^5 + 11X + 12 = (X + 1)(X^2 + 2X + 3)(X^2 - 3X + 4)$$

$$X^5 + 19X + 60 = (X^2 + 2X + 5)(X^3 - 2X^2 - X + 12)$$

$$X^5 + 29X + 15 = (X^2 + 3X + 5)(X^3 - 3X^2 + 4X + 3)$$

$$X^5 + 31X + 56 = (X^2 - 4X + 7)(X^3 + 4X^2 + 9X + 8).$$

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(Reçu le 7 avril 1990)

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