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Then the spectral resolution of  $L^2(F)$  reads

(7)

$$g(n) = \sum_{i=1,2} \langle g, u_i \rangle u_i(n) + 2\pi \int_0^\pi \langle g, \tilde{f}_\theta \rangle \tilde{f}_\theta(n) \frac{d\theta}{(q-1)^2 + 4q \sin^2 \theta}.$$

We end this paper by showing that, as one might expect from the theory of Eisenstein series, the eigenfunctions  $f_\lambda$  can be parametrized as a family of functions that depend holomorphically on a complex parameter. Precisely, let

$$E(n, s) = q^{ns}(q^{s-1} - q^{1-s}) + q^{n(1-s)}(q^s - q^{-s}).$$

Then  $E(n, s)$  is entire in  $s$  and satisfies the functional equation

$$E(n, s) = -E(n, 1-s).$$

Furthermore, a direct computation shows that

$$(TE)(n, s) = (q^s + q^{1-s})E(n, s).$$

There are two ways in which  $\lambda = q^s + q^{1-s}$  can be real. Write  $s = \sigma + it$ .

If  $t = \frac{k\pi}{\log q}$  then  $\lambda = (-1)^k(q^\sigma + q^{1-\sigma})$ , and in particular

$$\lambda = q + 1 (\lambda = -(q+1))$$

if  $\sigma = 1$  and  $k$  is even ( $k$  is odd). Otherwise we must have  $\sigma = \frac{1}{2}$ . If we write

$t = \frac{\theta}{\log q}$  we obtain our  $\lambda = 2\sqrt{q} \cos \theta$ .

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