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$$b + jc \not\equiv 0 \pmod{q-1} \quad \text{for all } j \quad \text{with} \quad 0 \leq j \leq n-1,$$

in view of [10, Lemmas 2.1 and 2.2]. Assume also that

$$c \not\equiv 0 \pmod{q-1},$$

since the result has been proved in [5] for $c \equiv 0 \pmod{q-1}$.

Theorem 1.1 is clear for $n = 1$, so let $n > 1$ and assume as induction hypothesis that

$$S_{n-1}(a+c, b+c, c) = P_{n-1}(a+c, b+c, c).$$

By (3.8) and (3.12), if $d \nmid n$,

$$\begin{aligned} S_n(a, b, c) &= P_{n-1}(a+c, b+c, c) \frac{G(a)G(b)G(cn)\bar{G}(a+b+(n-1)c)}{qG(c)} \\ &= P_n(a, b, c), \end{aligned}$$

whereas

$$S_n(a, b, c) + (q-1)P_n(a, b, c) = qP_n(a, b, c), \quad \text{if } d \mid n.$$

Thus $S_n(a, b, c) = P_n(a, b, c)$ in both cases, proving Theorem 1.1. The proofs of Theorems 1.1a and 1.1b follow similarly, from (3.8a), (3.12a) and (3.8b), (3.12b) in place of (3.8), (3.12).

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