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$$b + jc \not\equiv 0 \pmod{q-1} \quad \text{for all } j \quad \text{with} \quad 0 \leq j \leq n-1,$$

in view of [10, Lemmas 2.1 and 2.2]. Assume also that

$$c \not\equiv 0 \pmod{q-1},$$

since the result has been proved in [5] for $c \equiv 0 \pmod{q-1}$.

Theorem 1.1 is clear for $n = 1$, so let $n > 1$ and assume as induction hypothesis that

$$S_{n-1}(a+c, b+c, c) = P_{n-1}(a+c, b+c, c).$$

By (3.8) and (3.12), if $d \nmid n$,

$$\begin{aligned} S_n(a, b, c) &= P_{n-1}(a+c, b+c, c) \frac{G(a)G(b)G(cn)\bar{G}(a+b+(n-1)c)}{qG(c)} \\ &= P_n(a, b, c), \end{aligned}$$

whereas

$$S_n(a, b, c) + (q-1)P_n(a, b, c) = qP_n(a, b, c), \quad \text{if } d \mid n.$$

Thus $S_n(a, b, c) = P_n(a, b, c)$ in both cases, proving Theorem 1.1. The proofs of Theorems 1.1a and 1.1b follow similarly, from (3.8a), (3.12a) and (3.8b), (3.12b) in place of (3.8), (3.12).

REFERENCES

- [1] ANDERSON, G. W. The evaluation of Selberg sums. *Comptes Rendus Acad. Sci. Paris* 311, Série I (1990), 469-472.
- [2] ANDERSON, G. W. A short proof of Selberg's generalized beta formula. *Forum Math.* 3 (1991), 415-417.
- [3] ASKEY, R. Some basic hypergeometric extensions of integrals of Selberg and Andrews. *SIAM J. Math. Anal.* 11 (1980), 938-951.
- [4] ASKEY, R. and D. RICHARDS. Selberg's second beta integral and an integral of Mehta. In *Probability, Statistics, and Mathematics*, T. W. Anderson *et al.*, eds., pp. 27-39, Academic Press, Boston, MA, 1989.
- [5] AUTUORE, J. and R. EVANS. Evaluations of Selberg character sums. In *Analytic Number Theory*, B. C. Berndt *et al.*, eds., pp. 13-21, Birkhäuser, Boston, MA, 1990.
- [6] CELNIKER, N., S. POULOS, A. TERRAS, C. TRIMBLE and E. VELASQUEZ. Is there life on finite upper half planes? (To appear.)
- [7] EVANS, R. Identities for products of Gauss sums over finite fields. *Enseignement Math.* 27 (1981), 197-209.
- [8] —— A character sum for root system G_2 . *Proc. Amer. Math. Soc.*, (to appear).

- [9] EVANS, R., J. PULHAM and J. SHEEHAN. On the number of complete subgraphs contained in certain graphs. *J. Combin. Theory (Series B)* 30 (1981), 364-371.
- [10] EVANS, R. and W. ROOT. Conjectures for Selberg character sums. *J. Ramanujan Math. Soc.* 3 (1) (1988), 111-128.
- [11] GASPER, G. and M. RAHMAN. *Basic hypergeometric series*. Cambridge, NY, 1990.
- [12] GREENE, J. and D. STANTON. A character sum evaluation and Gaussian hypergeometric series. *J. Number Theory* 23 (1986), 136-148.
- [13] KOBBLITZ, N. The number of points on certain families of hypersurfaces over finite fields. *Compositio Math.* 48 (1983), 3-23.
- [14] KOIKE, M. Hypergeometric series over finite fields and Apery numbers. (To appear.)

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