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$$(**) \quad \Delta_K(t) = \Delta_{\bar{K}}(t)^2(t^{(1-\lambda)/2} + \dots + 1 + \dots + t^{(\lambda-1)/2}) + 2f(t),$$

where $f(t) \in \mathbf{Z}[t, t^{-1}]$ satisfies $f(t) = f(t^{-1})$. Writing

$$f(t) = a_n t^{-n} + \dots + a_0 + \dots + a_n t^n,$$

we see $f(1) \equiv f(-1) \pmod{4}$. Since $\Sigma \rightarrow \Sigma/G$ is a 2-fold cover branched over K , $|\Delta_K(-1)| = |H_1(\Sigma)| = 1$. So $1 = \Delta_K(1) \equiv \Delta_K(-1) \pmod{4}$, and we see $\Delta_K(-1) = 1$. Take equation (**), and plug in $t = 1$ and $t = -1$:

$$1 = 1 \cdot \lambda + 2 \cdot f(1)$$

$$1 = 1 \cdot (-1)^{(\lambda-1)/2} + 2 \cdot f(-1).$$

Thus $\lambda \equiv (-1)^{(\lambda-1)/2} \pmod{8}$ so $\lambda \equiv \pm 1 \pmod{8}$.

Applying the high-dimensional version of Murasugi's congruence one sees that if $G \times H \cong C_2 \times C_2$ acts on a homology q -sphere Σ so that Σ^G is a homology $q - 2$ sphere and Σ^H is a circle disjoint from Σ^G , then their linking number λ is congruent to ± 1 modulo 8. This and considerations from L -theory lead us to conjecture that if $G \times H \cong C_2 \times C_2$ acts on a homology q -sphere Σ so that Σ^G is a homology k -sphere and Σ^H is a homology $q - k - 1$ -sphere disjoint from Σ^G , then their linking number λ is congruent to ± 1 modulo 8.

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REFERENCES

- [C] da CRUZ, R. N. *Periodic knots*. Thesis (New York University 1987).
- [D-D] DAVIS, J. F. and T. TOM DIECK. Some exotic dihedral actions on spheres. *Indiana Univ. Math. J.* 37 (1988), 431-450.
- [F] FLOYD, E. On periodic maps and the Euler characteristics of associated spaces. *Trans. Amer. Math. Soc.* 72 (1952), 138-147.
- [H] HILLMAN, J. A. New proofs of two theorems on periodic knots. *Arch. Math. (Basel)* 37 (1981), 457-461.

- [L] LEVINE, J. A method for generating link polynomials. *Amer. J. Math.* 89 (1967), 69-84.
- [Mac] MAC LANE, S. *Homology* (Springer-Verlag 1963).
- [Mi] MILNOR, J. Infinite cyclic covers. *Conf. on topology of manifolds*, ed. J. G. Hocking (Prindle, Weber, and Schmidt, 1968), 115-133.
- [Mu] MURASUGI, K. On periodic knots. *Comment. Math. Helv.* 46 (1971), 162-174.

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