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Artikel: PERIODIC KNOTS, SMITH THEORY, AND MURASUGI'S CONGRUENCE

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$$(**) \qquad \Delta_K(t) = \Delta_{\bar{K}}(t)^2(t^{(1-\lambda)/2} + \dots + 1 + \dots + t^{(\lambda-1)/2}) + 2f(t)\,,$$

where $f(t) \in \mathbf{Z}[t, t^{-1}]$ satisfies $f(t) = f(t^{-1})$. Writing

$$f(t) = a_n t^{-n} + \dots + a_0 + \dots + a_n t^n\,,$$

we see $f(1) \equiv f(-1) \pmod 4$. Since $\Sigma \to \Sigma/G$ is a 2-fold cover branched over K, $|\Delta_K(-1)| = |H_1(\Sigma)| = 1$. So $1 = \Delta_K(1) \equiv \Delta_K(-1) \pmod 4$, and we see $\Delta_K(-1) = 1$. Take equation (**) and plug in $t = 1$ and $t = -1$:

$$1 = 1 \cdot \lambda + 2 \cdot f(1)$$
$$1 = 1 \cdot (-1)^{(\lambda-1)/2} + 2 \cdot f(-1)\,.$$

Thus $\lambda \equiv (-1)^{(\lambda-1)/2} \pmod 8$ so $\lambda \equiv \pm 1 \pmod 8$.

Applying the high-dimensional version of Murasugi's congruence ones sees that if $G \times H \cong C_2 \times C_2$ acts on a homology q-sphere Σ so that Σ^G is a homology $q - 2$ sphere and Σ^H is a circle disjoint from Σ^G, then their linking number λ is congruent to ± 1 modulo 8. This and considerations from L-theory lead us to conjecture that if $G \times H \cong C_2 \times C_2$ acts on a homology q-sphere Σ so that Σ^G is a homology k-sphere and Σ^H is a homology $q - k - 1$-sphere disjoint from Σ^G, then their linking number λ is congruent to ± 1 modulo 8.

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