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Combining this with 4.5 yields

$$\mathbf{n}(\tau(f)) - \tau(\mathbf{n}(f)) = [\mathbf{n}, \tau](f) = \kappa_1 \tau(f) + \mu_1 \mathbf{n}(f) = \kappa_1 \tau(f) + \mathbf{n} \tau(f)$$

which gives 4.6 as required.

§5. A GENERALIZED FOUR VERTEX THEOREM

The curves of constant curvature in the round 2-sphere S^2 , the upper half plane H^2 (hyperbolic space), and the Euclidean plane \mathbf{R}^2 are just the circles. Moreover, the stereographic projection $p: S^2 \rightarrow \mathbf{R}^2$ and the inclusion $i: H^2 \rightarrow \mathbf{R}^2$ both preserve these circles. Thus theorem 4.2 says that our form

$$\omega = \sqrt{|\kappa'(s)|} ds$$

along a curve γ in \mathbf{R}^2 pulls back via p or i to the form

$$\omega = \sqrt{|\kappa'_g(s)|} ds$$

along the corresponding curve γ' , where here $\kappa_g(s)$ and s refer to the geodesic curvature and arc-length of γ' in the metric for S^2 or H^2 . Thus we obtain the four vertex theorem for S^2 and H^2 . It follows that the four vertex theorem holds for all complete simply connected Riemannian surfaces of constant curvature. Finally if γ is a null-homotopic smooth simple closed curve on an arbitrary complete Riemannian surface M of constant curvature, then γ lifts one-to-one to a smooth simple closed curve with the same number of vertices on the simply connected universal cover of M . Once again it follows that the number of vertices is at least four.

Remark 5.1. Interestingly, simple closed homotopically non-trivial curves in the real projective plane always have at least three vertices [17]. Note that in non-orientable surfaces the number of honest vertices of a closed curve need not necessarily be even, since here geodesic curvature is only defined up to a sign.

§6. NORMAL FORM AND INVERSIVE CURVATURE

Let p be a non-vertex point of an oriented curve γ . Since the subgroup of Euclidean motions in G acts transitively on the points of \mathbf{R}^2 and the unit tangent vectors at these points, we may assume that the point $p \in \gamma$ which