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Proof. Let $g \in G$ such that $x = gSL(n, \mathbf{Z})$ and let $\Lambda = g\mathbf{Z}^n$. In view of Lemma A.2 a) there exists $\sigma > 0$ such that $d(\Delta) > \sigma$ for all subgroups Δ of Λ. Hence by Theorem A.1 there exists $\delta > 0$ such that for any $T \geqslant 0$ there exists a $s \geqslant T$ for which $\| u_s \xi \| \geqslant \delta$ for all $\xi \in \Lambda - \{0\}$. Let $K = \{hSL(n, \mathbf{Z}) \mid \| hp \| \geqslant \delta \text{ for all } p \in \mathbf{Z}^n - \{0\}\}$. Then by the Mahler criterion, recalled earlier, K is a compact subset of $SL(n, \mathbf{R})/SL(n, \mathbf{Z})$. From the choices it is clear that $\{s \geqslant 0 \mid u_s x \in K\}$ is an unbounded subset. This proves the theorem.

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