Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 36 (1990)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE POMPEIU PROBLEM REVISITED

Autor: Bagchi, S. C. / Sitaram, A.

Kapitel: 8. CONCLUDING REMARKS

DOI: https://doi.org/10.5169/seals-57903

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Mehr erfahren

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. En savoir plus

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. Find out more

Download PDF: 02.12.2025

ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch

then the argument is as in Theorem 7.1 and we can prove $\Pi_{\lambda}(1_E) = 0$. If Π_{λ} is not irreducible, then depending on n, m we can find a discrete series representation or an irreducible finite dimensional representation Π occurring either as a subrepresentation or as a subquotient of Π_{λ} for which $\Pi(1_E) = 0$. To do this, we need the exact G-module structure of Π_{λ} which in the case of $PSL(2, \mathbf{R})$ is available (see for example [14]).

§ 7.3. Consider the case $G = \mathbb{R}^2$ and G acting on itself by translations. In this case, Brown, Schreiber and Taylor have proved that there are no Pompeiu sets ([12]). In view of this it would be natural to ask if there are sets E satisfying the conditions of Theorem 7.1 at all. Identify \mathbb{R}^2 with G/K where G = M(2) and $K = SO(2, \mathbb{R})$; if $E \subseteq G/K$ then one can show that the condition of Brown, Schreiber and Taylor considered in Section 3 is equivalent to the condition $\Pi_{\lambda}(1_{\tilde{E}}) \neq 0$ for $\lambda \in \mathbb{C}$, $\lambda \neq 0$. (A special case of this observation is also made in [30]). Hence by the discussion in Section 3, there are plenty of sets E with this property. As we have seen, topologically $G \approx \mathbb{R}^2 \times SO(2, \mathbb{R})$. We now observe that if E is chosen as above in \mathbb{R}^2 and E is a suitably chosen arc in E for all E is chosen as above in E and E is a subset of E satisfies E and E for all E is a suitably chosen arc in E for all E is chosen as well as E and E for all E is invariant. We need only to choose E whose length is irrational modulo E is irrational modulo E and E is a suitable chosen.

8. Concluding remarks

In this paper, we have restricted our attention to the Pompeiu property for a single set E. One can also consider the Pompeiu property for a collection of sets or distributions of compact support as in [9], [12]. There are also closely related properties such as the Morera property — see [12] for details.

As pointed out earlier the Pompeiu problem becomes easier if one considers only integrable functions. Investigations under this assumption have been done, for example, in [2], [20], [24] and [28]. If one only considers integrable functions one need not restrict oneself to relatively compact sets. Moreover, considering integrable functions is equivalent to considering finite complex measures. Thus for G a locally compact abelian group a Borel subset $E \subseteq G$ is said to be a determining set for finite complex measures if for a finite complex measure μ on G, $\mu(gE) = 0$ for all $g \in G$ implies $\mu = 0$.

For locally compact abelian groups it is easy to see that a set of finite Haar measure is a determining set for finite complex measures if and only if the Fourier transform $\hat{1}_E$ does not vanish on any nonempty open subset of the dual group \hat{G} . Thus bounded Borel subsets of \mathbb{R}^n of positive Lebesgue measure are determining sets by the analyticity of $\hat{1}_E$. Classical quasianalyticity results apply to give conditions on the growth of an unbounded subset $E \subseteq \mathbb{R}^n$ to be a determining set. Settling a problem that was open for some time, Kargaev ([17]) proved the existence of sets $E \subseteq \mathbb{R}^n$ of finite Lebesgue measure which are not determining sets for finite complex measures.

The problem of determining sets has also been studied with the class of probability measures replaced by other classes of measures, e.g., a class of infinite measures with growth/decay conditions (see [22], [11] and [28]). Also different groups of homeomorphisms acting on X have been considered in these studies.

Finally, we refer to the following form of the *support problem* analogous to the well known problem in the case of Radon transform. Let X be a symmetric space (Euclidean, compact or non-compact). Let x_0 be a fixed point of X. If E is a relatively compact subset of positive measure and if $\int_{gE} f = 0$ for all $g \in G$ with $d(x_0, gx_0) > R$ what can one say about the support of f with respect to the reference point x_0 ? (Here, f stands for the geodesic distance.) Some partial answers to this question are known (see [26] and [28]).

We have not addressed ourselves in this paper to the situation when X is an infinite-dimensional Hilbert space or X is an arbitrary Riemannian manifold. Another important problem we have not considered is the *local version* of the Pompeiu problem. (For this, we refer the reader to [5] and [6]). We have restricted ourselves to the situation of symmetric spaces and locally compact groups and the relationship of the Pompeiu problem with harmonic analysis and representation theory.

Acknowledgement. The authors thank Lawrence Zalcman for pointing out some errors in the original draft of this paper. They would also like to express their gratitude to K. R. Parthasarathy for initially getting them interested in the Pompeiu problem.

REFERENCES

^[1] BAGCHI, S. C. and A. SITARAM. Spherical mean periodic functions on semisimple Lie groups. *Pacific J. Math.* 84 (1979), 241-250.

^[2] BAGCHI, S. C. and A. SITARAM. Determining sets for measures on \mathbb{R}^n . Illinois J. Math. 26 (1982), 419-422.