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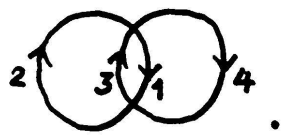
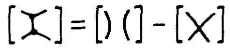
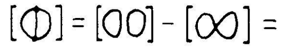


There are many instances of this sort of expansion outside of the theory of knots and links. For example, the following expansion (compare [78]) for *trivalent plane graphs G*



gives states that are locally four-valent plane graphs. If the value of a state S is taken to be *three raised to the number of crossing circuits in* S, then $[G]$ is the number of colorings of the edges of G with three colors so that three distinct colors meet at each vertex of G. The existence of such a coloring for a trivalent plane graph is well known to be equivalent to finding a four-coloring of its faces so that no two faces that share an edge receive the same color. It is a delicate matter to determine when $[G]$ is non-zero.

 $= 3^2 - 3 = 6\,.$

Other conventions, more closely related to tensor formalisms are discussed in [78] and [58].

In general, these pictorial expansions are a way to express the vertex weights of a model in a fashion that is easy to relate with the geometry of the diagrams themselves.

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