Appendix on state model formalism

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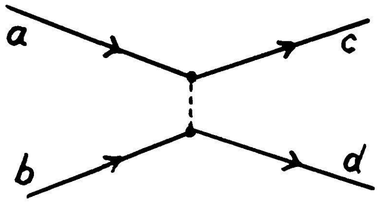
Then sums of vertex weights appear in the exponents. These divide into weights from 2-vertices (corresponding to curvature) and weights from interactions. The interaction weights depend upon angle and local spins — all information that is available on projecting in a given direction e.

This suggests the form of a model as

$$<K> = \int_{\sigma} d\sigma \exp\left(\iint_{K\times K} [p, q \mid \sigma]\,\mathcal{G} + \int_{K} [p \mid \sigma]\right)$$

where the problem of taking the limit over subdivisions of the space curve, and the definition of the limiting state space is certainly unsolved.

The proposed model is designed as a generalization of the planar models. The crossings are replaced by pairs of points on the curve, and the Gauss kernel appears, as in the linking number. In a piecewise-linear approximation to the model, the space curve is divided into straight-line segments. A state assigns a spin to each segment. If spin remains unchanged at a vertex, then the vertex contributes a simple angular term, as in the planar case. *If spin changes at a vertex, then this vertex must be paired up with another vertex so that the pair can be regarded as under-going a spin preserving interaction.*



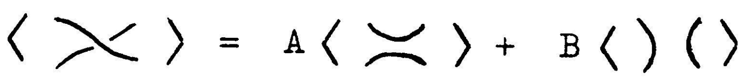
$a + b = c + d$

This is the generalization of the crossing in the planar case. A state is admissible if it is configured with such self-interactions allowing spin conservation. The three-dimensional approximation sums over all such admissible states.

APPENDIX ON STATE MODEL FORMALISM

This appendix is a short note on the formalism I use for expressing state models.

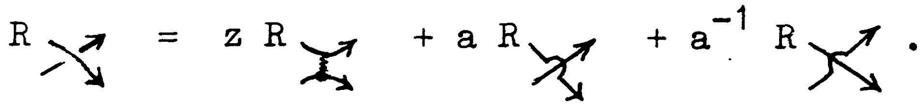
The bracket polynomial [41] is defined via equations of the form



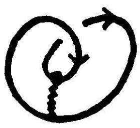
$\langle OK \rangle = d \langle K \rangle$

(It is a regular isotopy invariant when $B = (1/A)$ and $-d = A^2 + A^{-2}$.) Here the small diagrams stand for parts of larger diagrams that differ only as shown in the small diagrams. Since all diagrams are of the same type (unoriented link diagrams) these equations are easily understood.

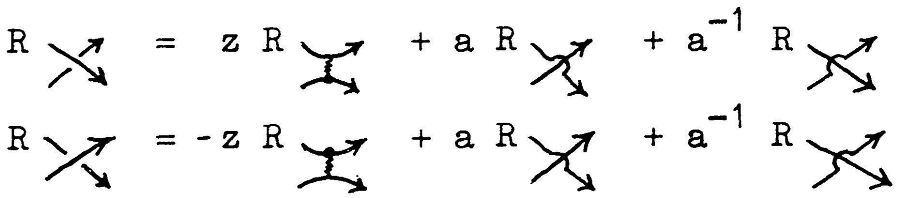
On the other hand we have used formalisms in this paper of the ilk



Here the small diagrams on the right-hand side of the equation replace a knot-diagrammatic crossing. The large diagrams on the right go out of the category of link diagrams to an appropriate category of labelled graphs. As a result (for example), one may legitimately ask: What is the value of the R-polynomial on the "mixture" shown below?

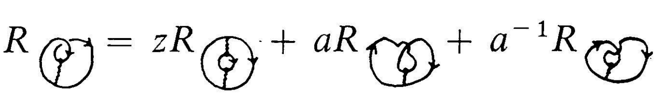


To answer this type of question in all cases I take the following point of view: *Equations such as*



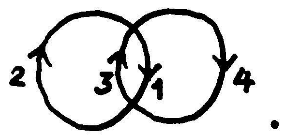
are a recursive algorithm for expressing R *as a sum of evaluations of decorated graphs – all of whose knot-theoretic crossings have been replaced by one of the choices in these equations.* A further rule must be given to specify the values of the decorated graphs (the "states") produced by this algorithm. (In the case of the R-polynomial this rule involves using the template (Section 2) to determine if the graph is admissible. If so, then its value is $\delta^{|S|-1}$ where $|S|$ is the number of circuits in the state S.)

Thus

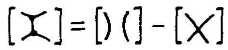


$$= z\delta + a^{-1}$$

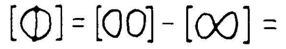
for the template



There are many instances of this sort of expansion outside of the theory of knots and links. For example, the following expansion (compare [78]) for *trivalent plane graphs G*



gives states that are locally four-valent plane graphs. If the value of a state S is taken to be *three raised to the number of crossing circuits in* S, then $[G]$ is the number of colorings of the edges of G with three colors so that three distinct colors meet at each vertex of G. The existence of such a coloring for a trivalent plane graph is well known to be equivalent to finding a four-coloring of its faces so that no two faces that share an edge receive the same color. It is a delicate matter to determine when $[G]$ is non-zero.

 $= 3^2 - 3 = 6\,.$

Other conventions, more closely related to tensor formalisms are discussed in [78] and [58].

In general, these pictorial expansions are a way to express the vertex weights of a model in a fashion that is easy to relate with the geometry of the diagrams themselves.

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