

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 36 (1990)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: STATE MODELS FOR LINK POLYNOMIALS
Autor: Kauffman, Louis H.
Kapitel: VIII. Applications and questions
DOI: <https://doi.org/10.5169/seals-57900>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 19.08.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

A particularly nice model occurs for index set in the form

$$\{-n, -n+2, \dots, n-2, n\} \quad \text{with} \quad \lambda = w.$$

This gives a series of one-variable specializations of the Homfly polynomial. (See [40], [58], [93].) *Is there a Yang-Baxter model for the full Homfly polynomial?* This is an open question.

A similar approach works for the Dubrovnik form of the Kauffman polynomial. See [58], [93]. The expansion formula has the appearance.

$$[\text{X}] = z [\text{I}] - z [\text{H}] + w [\text{I}] + w^{-1} [\text{H}] + [\text{X}]$$

(It is understood that reversing the orientation of a line is accompanied by the negation of its spin.) Once again, the dot on a line means that it has smaller spin.

VIII. APPLICATIONS AND QUESTIONS

This section is devoted to a few applications of the skein and state models and related questions.

1. Let ∇_K denote the Conway polynomial. The skein model is embodied in the formula of section 6:

$$\nabla_K = \sum_{L, |L|=1} (-1)^{t-(L)} z^{t(L)}$$

from which we see easily that

$$\max \deg \nabla_K \leq V - S + 1 = \rho(K)$$

where V is the number of crossings in the diagram K , S is the number of Seifert circuits (the set of circuits obtained by splicing all crossings of K). One knows that $\rho(K) = \text{rank}(H_1(F))$ where F is the Seifert spanning surface [42] corresponding to the diagram K . If K is an alternating link then $\max \deg \nabla_K = \rho(K)$ [76]. This is generalized to the class of alternative links in [42], using the FKT model. Is there a proof using the skein model?¹⁾

In the case where all the crossings are of positive type, we see from the skein model that all terms of ∇_K are positive, and it is then easy to see that the highest degree term is of degree $\rho(K)$.

¹⁾ Note added in proof: A proof using the skein model for the theorem on alternative links has been found by John Mathias — University of Maryland, May 1989.

2. Similar remarks apply to the Homfly model of section 3. In the case of the Yang-Baxter model for the Homfly polynomial given in section 7, it is easy to see that the highest z -degree is $\rho(K)$ when K is positive — this time by constructing an appropriate spin state.
3. Thistlethwaite [89] proves that the writhe $w(K)$ is an ambient isotopy invariant for K alternating and reduced. It would be useful to see a proof of this result using the skein model for D_K (section 4).
4. The Alexander polynomial Δ_K is given by the formula

$$\begin{aligned}\Delta_K(t) &\doteq \nabla_K(\sqrt{t} - 1/\sqrt{t}) \\ &= \sum_{|L|=1} (-1)^{t^-(L)} (\sqrt{t} - 1/\sqrt{t})^{t(L)}\end{aligned}$$

where \doteq denotes equality up to sign and powers of t . One knows ([23]) that if K bounds a smooth disk in the upper 4-space $((x, y, z, t)$ with $t > 0$) then

$$\Delta_K(t) \doteq f(t)f(t^{-1})$$

for some polynomial $f(t)$. Can this fact be deduced directly from the skein model or from the FKT model? A solution should generalize to give new information about the full skein polynomial behaviours on slice links.

IX. RELATIONS WITH MATHEMATICAL PHYSICS

I have deliberately included a description of the Yang-Baxter models in this paper in order to raise the question of the relation of the skein models to mathematical physics. The Yang-Baxter models can be regarded as averages of scattering amplitudes over all possible spin states — hence as discrete Feynman integrals, or as partition functions for two-dimensional statistical mechanics models. The FKT model for the Conway polynomial can be seen [57] as the low temperature limit of a partition function of a generalized Potts model.

META-TIME

If we interpret the FKT model or the skein models in a particle interaction framework, then a curious and interesting issue arises:

Think of a particle moving forward and backward in “time” on a given universe. The “same” particle may traverse a given site (crossing) twice.