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**Autor:** Kauffman, Louis H.  
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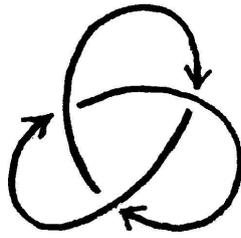
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is admissible and contributes  $(-z)^3\delta$  to the summation for the negative trefoil



IV. A SKEIN MODEL FOR THE KAUFFMAN POLYNOMIAL

The work of section 3 goes over essentially verbatim for the Dubrovnik version of the Kauffman polynomial. Recall that in this context a template is obtained by first orienting the edges of the universe  $U$  underlying the unoriented  $K$ , and then labelling the edges of  $U$ .

The chart in Figure 7 shows the cases of admissible splices at crossings (with respect to the skein template algorithm). Each splice has been labelled with its corresponding vertex weight. Note that a splice is admissible if it indicates the form of passage that is obtained from an approach to the crossing that meets it as an under-crossing. Such approaches give active crossings in the skein template algorithm.

I have retained only the arrow for the first passage after each split, because the orientation on the other edge may change under the direction of the template. *The crossings are oriented because each end-node (unlink) produced by the skein template algorithm acquires an orientation from the directions of travel given by the template.*

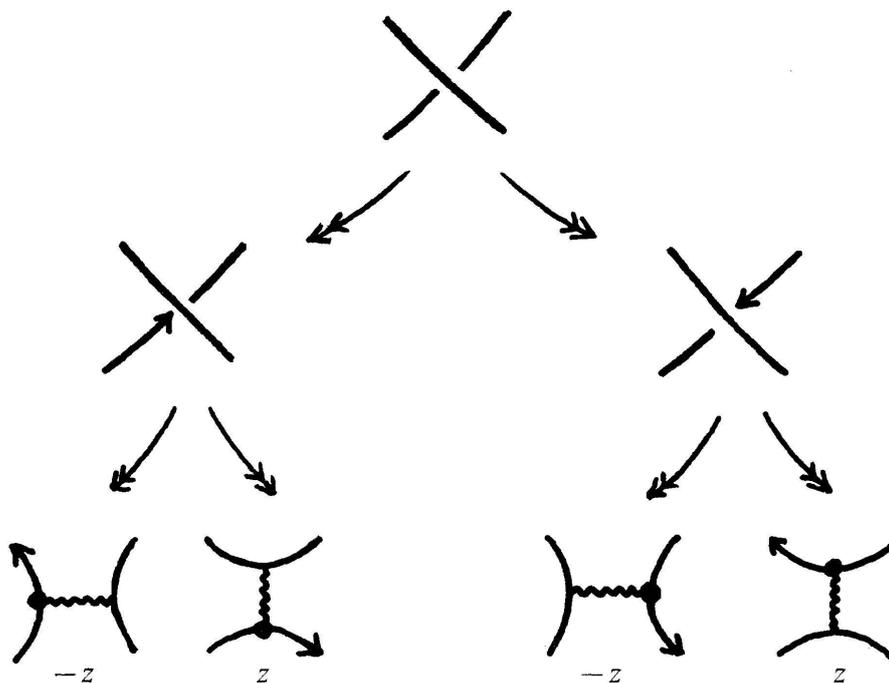
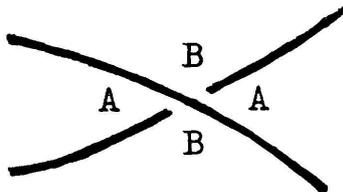


FIGURE 7

In order to understand the pattern of these admissible splices, consider an unoriented crossing



I have labelled two of its regions  $A$ . These are swept out by turning the overcrossing line counterclockwise. The other two regions are labelled  $B$ . The  $A$ -splice of the crossing is that splice that joins the two regions labelled  $A$ . The  $B$ -splice joins the two regions labelled  $B$ .

We then see that a passage is admissible in the  $A$ -splice if it occurs on the *right* for an observer who stands in between the strands, facing in the direction of the passage from basepoint. Similarly, the admissible  $B$ -splices are on the left for such an observer.

Call an admissible splice *negative* if it is of  $B$ -type. (This receives a  $(-z)$  in Figure 7.)

With these definitions we have

$$t(L) = \text{number of splices to obtain } L \text{ from } K .$$

$$t_-(L) = \text{number of negative splices .}$$

$Au(K, T)$  = set of admissible unlinks relative to  $K$  and  $T$  (here  $K$  is unoriented).

Then the Dubrovnik polynomial is given by the formula

$$D_K = \sum_{L \in Au(K, T)} (-1)^{t_-(L)} z^{t(L)} a^{w(L)} \mu^{|L|-1}$$

$$(\mu = 1 + (a - a^{-1})/z) .$$

As a state-expansion we can write

$$D_{\times} = z(D_{\text{A-splice}} + D_{\text{B-splice}}) - z(D_{\text{A-splice}} + D_{\text{B-splice}})$$

$$+ a(D_{\text{A-splice}} + D_{\text{A-splice}} + D_{\text{A-splice}} + D_{\text{A-splice}})$$

$$+ a^{-1}(D_{\text{A-splice}} + D_{\text{A-splice}} + D_{\text{A-splice}} + D_{\text{A-splice}}) .$$

Once again,

$$D_K = \sum_{L \in Au(K, T)} \langle K | L \rangle \mu^{|L|-1}$$

where  $\langle K | L \rangle$  denotes the product of vertex weights (all relative to the given template). Independence of the template follows from the well-definedness of the polynomial itself.

*Remark.* It would be very interesting to know the relationship between this state model for the Kauffman polynomial and the extraordinary model of Jaeger [34]. Jaeger gives a state expansion where the states are a collection of oriented knots and links. Each state is itself evaluated via the regular isotopy version of the Homfly polynomial.

## V. GRAPH POLYNOMIALS

The two skein polynomials (Homfly and Kauffman) each have three variable extensions to rigid vertex isotopy invariants of 4-valent graphs imbedded in three-space. This construction has been announced in [45]. (See also [56] and [74].) Our skein models involve 4-valent graphs implicitly, and so give rise to a natural definition for these extended polynomials as state models.

Let the new variables  $A$  and  $B$  be given, with  $z = A - B$  the usual  $z$  for the skein polynomials. The extended polynomials are then defined by the axioms:

### HOMFLY EXTENSION AXIOMS

1.  $R \begin{array}{c} \nearrow \\ \searrow \end{array} = AR \begin{array}{c} \curvearrowright \\ \curvearrowleft \end{array} + R \begin{array}{c} \nearrow \\ \searrow \end{array} \times \begin{array}{c} \nearrow \\ \searrow \end{array},$   
 $R \begin{array}{c} \searrow \\ \nearrow \end{array} = BR \begin{array}{c} \curvearrowright \\ \curvearrowleft \end{array} + R \begin{array}{c} \searrow \\ \nearrow \end{array} \times \begin{array}{c} \searrow \\ \nearrow \end{array},$
2.  $R_K =$  usual regular isotopy Homfly polynomial if  $K$  is free of graphical vertices ( $\begin{array}{c} \nearrow \\ \searrow \end{array} \times \begin{array}{c} \nearrow \\ \searrow \end{array}$ ).

### KAUFFMAN EXTENSION AXIOMS

1.  $D \begin{array}{c} \nearrow \\ \searrow \end{array} \times \begin{array}{c} \searrow \\ \nearrow \end{array} = AD \begin{array}{c} \curvearrowright \\ \curvearrowleft \end{array} + BD \begin{array}{c} \curvearrowleft \\ \curvearrowright \end{array} + D \begin{array}{c} \nearrow \\ \searrow \end{array} \times \begin{array}{c} \searrow \\ \nearrow \end{array},$
2.  $D_K =$  usual regular isotopy Dubrovnik polynomial if  $K$  is free of graphical vertices ( $\begin{array}{c} \nearrow \\ \searrow \end{array} \times \begin{array}{c} \searrow \\ \nearrow \end{array}$ ).