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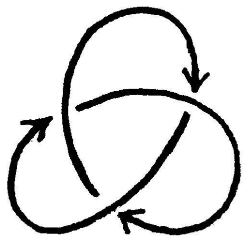
is admissible and contributes $(-z)^3\delta$ to the summation for the negative trefoil



IV. A Skein Model for the Kauffman polynomial

The work of section 3 goes over essentially verbatim for the Dubrovnik version of the Kauffman polynomial. Recall that in this context a template is obtained by first orienting the edges of the universe U underlying the unoriented K, and then labelling the edges of U.

The chart in Figure 7 shows the cases of admissible splices at crossings (with respect to the skein template algorithm). Each splice has been labelled with its corresponding vertex weight. Note that a splice is admissible if it indicates the form of passage that is obtained from an approach to the crossing that meets it as an under-crossing. Such approaches give active crossings in the skein template algorithm.

I have retained only the arrow for the first passage after each split, because the orientation on the other edge may change under the direction of the template. *The crossings are oriented because each end-node (unlink) produced by the skein template algorithm acquires an orientation from the directions of travel given by the template.*

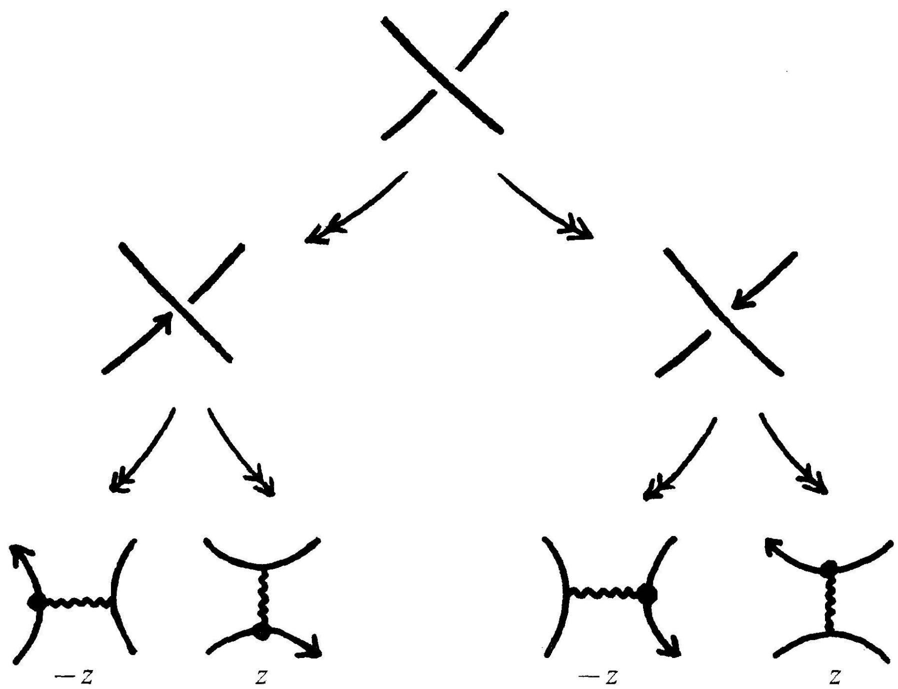
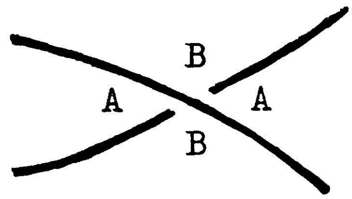


Figure 7

In order to understand the pattern of these admissible splices, consider an unoriented crossing



I have labelled two of its regions A. These are swept out by turning the overcrossing line counterclockwise. The other two regions are labelled B. The *A-splice* of the crossing is that splice that joins the two regions labelled A. The *B-splice* joins the two regions labelled B.

We then see that a passage is admissible in the A-splice if it occurs on the *right* for an observer who stands in between the strands, facing in the direction of the passage from basepoint. Similarly, the admissible B-splices are on the left for such an observer.

Call an admissible splice *negative* if it is of B-type. (This receives a $(-z)$ in Figure 7.)

With these definitions we have

$$t(L) = \text{number of splices to obtain } L \text{ from } K\,.$$

$$t_-(L) = \text{number of negative splices}\,.$$

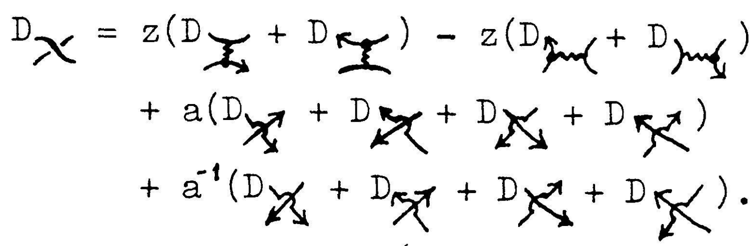
$\mathrm{Au}(K, T)$ = set of admissible unlinks relative to K and T (here K is unoriented).

Then the Dubrovnik polynomial is given by the formula

$$D_K = \sum_{L \in \mathrm{Au}(K,T)} (-1)^{t_-(L)} z^{t(L)} a^{w(L)} \mu^{|L|-1}$$

$$\left(\mu = 1 + (a - a^{-1})/z\right).$$

As a state-expansion we can write



Once again,

$$D_K = \sum_{L \in \mathrm{Au}(K,T)} \langle K \mid L \rangle\, \mu^{|L|-1}$$

where $<K \mid L>$ denotes the product of vertex weights (all relative to the given template). Independence of the template follows from the well-definedness of the polynomial itself.

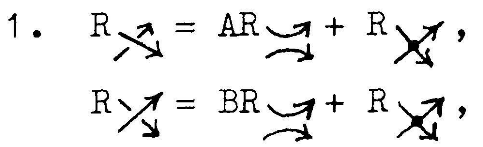
Remark. It would be very interesting to know the relationship between this state model for the Kauffman polynomial and the extraordinary model of Jaeger [34]. Jaeger gives a state expansion where the states are a collection of oriented knots and links. Each state is itself evaluated via the regular isotopy version of the Homfly polynomial.

V. Graph polynomials

The two skein polynomials (Homfly and Kauffman) each have three variable extensions to rigid vertex isotopy invariants of 4-valent graphs imbedded in three-space. This construction has been announced in [45]. (See also [56] and [74].) Our skein models involve 4-valent graphs implicitly, and so give rise to a natural definition for these extended polynomials as state models.

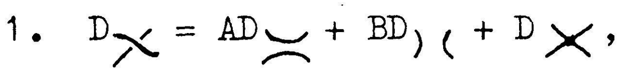
Let the new variables A and B be given, with $z = A - B$ the usual z for the skein polynomials. The extended polynomials are then defined by the axioms:

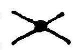
Homfly Extension Axioms



2. R_K = usual regular isotopy Homfly polynomial if K is free of graphical vertices ().

Kauffman Extension Axioms



2. D_K = usual regular isotopy Dubrovnik polynomial if K is free of graphical vertices ().