through the bibliographical compilations made by W. L. Schaaf on the recent literature on games, published by the NCTM.

Still more interesting is the fact that there is so much deep mathematics with the flavour of games. Among modern examples one can select a few in which this is particularly obvious. Some of them can in fact be used as a basis for amusing and entertaining games: the four colour theorem, Ramsey's theorem, Sperner's lemma, the triangular billiards table, Helly's theorem, Hadwiger's conjecture,...

After these considerations it becomes quite easy to understand why so many among the greatest mathematicians have been so fond of games and in some cases have become great practitioners of some of them.

Fermat made very extensive and deep investigations on magic squares.

Leibniz writes in a letter to De Montmort (1715): "*Men are never more ingenious than in the invention of games; the spirit finds himself there at his leisure... it would be desirable to have a whole course about games mathematically treated*".

Euler writes in a letter to Goldbach (1744): "*Here one plays much chess. Among others there is a Jew who plays extraordinarily well. I have taken lessons from him and now I am so far that I win him most of the games*". And one can easily perceive that a good part of Euler's research has the flavor of mathematical games.

Many names of famous mathematicians could be mentioned here in connection with the ludic spirit of mathematics: Fibonacci, Cardano, Pascal, Daniel Bernoulli, Gauss, Hamilton, Hilbert, Einstein, von Neumann,...

GAMES AND PUZZLES AS A MEANS FOR POPULARIZATION

After what we have seen about the similarity in their intrinsic nature, structure, spirit and ways of practice of games and mathematics it should be quite clear that:

a game is very often good mathematics and frequently leads to deep and important mathematical developments;

playing a good game or puzzle is the closest thing to doing mathematics; it stimulates essentially the same abilities and often requires the same type of strategies as mathematics;

mathematics can be approached with the same ludic spirit of games and has been thus approached by many of the greatest mathematicians; this ludic and aesthetic spirit is at the very root of mathematical creativity.

But a game can avoid many of the pitfalls of solemn and serious mathematical presentations:

Games do not need boring systematic introductions before arriving to something interesting, as is often the case with our presentation of mathematics.

Good games and puzzles can avoid the effect of the psychological blocks that straight mathematical presentations tend to cause not only in children, but also in many adults, very often because of previous unpleasant mathematical experiences.

Games place everybody initially in a situation of equality, in which not so much depends on previous performance and knowledge as in mathematics.

Games are better at fostering ingenuity, imagination, phantasy, experimentation, manipulation,... since they constitute much more clearly a free activity quite open to us.

All these reasons can explain the great success of the really good writers of mathematical recreations along many centuries, and one can easily agree with Berlekamp, Conway and Guy, when in the dedication of their excellent work on games [1] they write:

> "To MARTIN GARDNER who has brought more mathematics to more millions than anyone else".

There are, of course, good and bad games and puzzles. What are the characteristics of a stimulating game from our point of view? I will point out some of the general traits that in my opinion help a game to be good:

attractive, beautiful, with a flavour of nearness and handiness...

easy to start on, no need of long and systematic introductions, can be approached from zero;

reasonably challenging, maybe looking easier than it really is;

perhaps with deep consequences for the real world, for the sciences, for mathematics,...

rooted in the particular culture, history of the people;

extendable, perhaps a whole world behind it...

its presentation should be made with genuine and contagious enthusiasm.

The great master of popularization of mathematics in the second half of our century, Martin Gardner, has assessed the situation quite rightly, talking about the use of games for mathematical teaching. The same could be said for popularization in general: "Surely the best way to wake up a student is to present him with an intriguing mathematical game, puzzle, magic trick, joke, paradox, limerick or any of a score of other things that dull teachers tend to avoid because they seem frivolous" (*Mathematical Carnival,* Preface).

Summarizing one could say that mathematics is a great and sophisticated game that, besides, happens to be an intellectual work of art bearing at the same time an intense light to explore the universe and so having great practical repercussions. The attempts to popularize mathematics through its applications, its history, the biography of the most interesting mathematicians, through the relationships with philosophy or other aspects of the human mind can serve very well to let mathematics be known by many persons. But possibly no other method can convey what is the right spirit of doing mathematics better than a well chosen game.

Finally I would like to present some of the more common objections that one encounters when dealing with games related to the popularization and the teaching of mathematics.

"I hate mathematical games".

There are many people, among them also mathematicians, who seem to be confused about the whole matter. There is a certain kind of puzzle and questions that are open to a great amount of ambiguity and trickery, in many cases intended to show you quite clearly how stupid you are. Such tricks can become noxious and harmful and they can cause a certain inferiority complex if one is overexposed to them. They give rise to a very false image of mathematics. Good mathematical games are not so. They are as well defined as any honest mathematical problem and their effect is to stimulate your ingenuity and imagination, not to let you down.

"I find them completely useless and inconsequential".

A little knowledge of the historical evolution of mathematics shows, as I have pointed out before, that many profound ideas of the greatest mathematicians could be traced down to their involvement in this kind of ludic thinking. It is rather difficult to draw the line between the important and the inconsequential. When presenting mathematical games to the great public one should take some

care to convey with them a certain respect towards the implications this kind of thinking can have and has really had along the history of science.

"It is a waste of time".

There is a difference between a healthy curiosity and dedication and a manic obsession that can absorb you for weeks without end. It is a matter of temperance and balance.

"It takes me far from my field".

Who knows? Perhaps an ingenious idea in a game can help you approach one of your problems from a new viewpoint. Sometimes it is very advisable to make excursions into neighbouring and even into distant fields, particularly when we get stuck in our work.

"Mathematical games lead our students to think that our mathematical life is something light and easy. When they find out, they shrink and go to something else".

There are games requiring different levels of skill and effort. One can easily find authentic research open problems among them. Many games can strongly attract people and stimulate them to do very hard work.

"Applied and useful mathematics is the real, serious enterprise. Pure mathematics is sterile. Approaching mathematics as a game is catastrophic".

I would like to end this paper with a quotation from Piet Hein *(Grooks)* that summarizes the whole of it and answers pretty well to this objection:

Taking fun
as simply fun
and earnestness
in earnest
shows how thoroughly
thou none
of the two discernest