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It follows that

$$(5.10) \quad I(V_m, a; V_n, b) = \sum_{j=0}^{\min\{m, n\}} \prod_{i=0}^{j-1} \frac{a - b + \frac{1}{2}(m+n) - i}{a - b - \frac{1}{2}(m+n) + i} P_j .$$

We summarize our results in the following theorem.

**THEOREM 5.11.** *The R-matrix associated to the representation*

$$V = V_{m_1}(a_1) \otimes \cdots \otimes V_{m_k}(a_k)$$

of  $Y$  is given by

$$R(a-b) = \left( \prod_{i,j=1}^k I(V_{m_i}, a + a_i; V_{m_j}, b + a_j) \right) \sigma ,$$

where the intertwining operators are given by equation (5.10) and  $\sigma$  is the switch map. The order of the factors in the product is such that the  $(i, j)$ -term appears to the left of the  $(i', j')$ -term iff

$$i > i' \quad \text{or} \quad i = i' \quad \text{and} \quad j < j' .$$

## 6. CONCLUDING REMARKS

Since we have discussed only the Yangian associated to  $\mathfrak{sl}_2$  in this paper, it may be worth-while to indicate the extent to which the results above can be generalized to the Yangian  $Y(\mathfrak{a})$  associated to an arbitrary finite-dimensional complex simple Lie algebra  $\mathfrak{a}$ .

The definition of  $Y(\mathfrak{a})$  is precisely as in (1.1), except of course that  $\{I_\lambda\}$  should be an orthonormal basis of  $\mathfrak{a}$  with respect to some invariant inner product. The formulae

$$\tau_a(x) = x, \quad \tau_a(J(x)) = J(x) + ax,$$

for  $x \in \mathfrak{a}$ , again define a one-parameter group of Hopf algebra automorphisms of  $Y(\mathfrak{a})$ , and the relation, discussed in section 5, between solutions of the quantum Yang-Baxter equation and intertwining operators between tensor products of representations of  $Y(\mathfrak{a})$ , which follows from the existence of the  $\tau_a$ , is also valid in the general case.

The major complication which arises in the general case is that, except when  $\mathfrak{a} = \mathfrak{sl}_2$ , the assignment

$$x \rightarrow x, \quad J(x) \rightarrow 0$$

does not extend to a homomorphism of algebras

$$Y(\mathfrak{a}) \rightarrow U(\mathfrak{a}).$$

However, if  $\mathfrak{a} = \mathfrak{sl}_n$ , there is a substitute, namely

$$\varepsilon(x) = x, \quad \varepsilon(J(x)) = \frac{1}{4} \sum_{\lambda, \mu} \text{trace}(x(I_\lambda I_\mu + I_\mu I_\lambda)) I_\lambda I_\mu$$

(see [4]). One can now define evaluation representations of  $Y$  by pulling back representations of  $\mathfrak{sl}_n$  using the homomorphisms  $\varepsilon \circ \tau_a$ . We make the following conjecture, generalizing the case  $n = 2$  proved above:

**CONJECTURE.** *Every finite-dimensional irreducible representation of  $Y(\mathfrak{sl}_n)$  is isomorphic to a tensor product of evaluation representations.*

If  $\mathfrak{a} \neq \mathfrak{sl}_n$  for any  $n$ , it turns out that there is no homomorphism of algebras  $\varepsilon: Y(\mathfrak{a}) \rightarrow U(\mathfrak{a})$  such that  $\varepsilon(x) = x$  for all  $x \in U(\mathfrak{a})$  (see [4]), and there is no obvious analogue of the evaluation representations. However, there is a straightforward generalization of Theorem 2.4 classifying the finite-dimensional irreducible representations of  $Y(\mathfrak{a})$  in terms of highest weights. Such representations are now in one-to-one correspondence with  $l$ -tuples of monic polynomials  $\{P_i\}$ , where  $l = \text{rank } \mathfrak{a}$ . It is natural to define a fundamental representation  $V_{i,a}$  of  $Y(\mathfrak{a})$  to be one corresponding to a set of polynomials of the form

$$\begin{aligned} P_j(u) &= 1 \quad \text{if } j \neq i, \\ P_i(u) &= u - a \end{aligned}$$

for some  $a \in \mathbf{C}$ . As in the case of semisimple Lie algebras, it can be shown that every finite-dimensional irreducible representation of  $Y(\mathfrak{a})$  (for any  $\mathfrak{a}$ ) is a subquotient of a tensor product of fundamental representations (see [3]), and understanding the structure of the fundamental representations  $V_{i,a}$  is thus an important first step towards understanding the most general representation. The case where the  $i^{\text{th}}$  node of the Dynkin diagram is extremal (i.e. joined to only one other node), and also the case where the  $i^{\text{th}}$  fundamental representation of  $\mathfrak{a}$  is the adjoint representation, were discussed in [4], and the general case is dealt with in [3], except for a few nodes of  $E_7$  and  $E_8$  (some cases have also appeared without proof in the physics literature [10]).

As we have already mentioned, to any finite-dimensional irreducible representation of any Yangian  $Y(\mathfrak{a})$  is associated a rational solution of the QYBE. Many examples of such  $R$ -matrices have been computed in the literature (see [10] for a recent summary), although these calculations are mainly restricted to the case of those fundamental representations which are irreducible as representations of  $\mathfrak{a}$  (the only exception seems to be [11], which considers the case where  $i$  corresponds to the spin representation(s) of  $\mathfrak{a} = \mathfrak{so}_n$ ). In [3] we carry out the computations for most of the remaining fundamental representations, and also for the adjoint representation of  $Y(\mathfrak{sl}_n)$ . However, the computation of the  $R$ -matrix associated to an arbitrary representation of  $Y(\mathfrak{a})$  remains a difficult open problem.