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Artikel: AN ELEMENTARY PROOF OF A THEOREM ON QUADRATIC FORMS OVER THE RATIONAL NUMBERS

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3.2. PROPOSITION. *Let* $a, b, c \in \mathbf{Q}^\times$, $a, b, c > 0$. *Then*

$$\langle\langle a, b, c\rangle\rangle \cong \langle\langle 1, 1, 1\rangle\rangle = 8\langle 1\rangle .$$

Proof. Calculating in the Witt ring WF we have

$$\langle\langle a, b, 1\rangle\rangle \perp (-1)\langle\langle a, b, c\rangle\rangle = \langle\langle a, b\rangle\rangle (\langle 1, 1\rangle \perp (-1)\langle 1, c\rangle)$$
$$= \langle\langle a, b\rangle\rangle \langle 1, -c\rangle = \langle\langle a, b, -c\rangle\rangle = 0 \text{ by Proposition 3.1.}$$

Therefore $\langle\langle a, b, 1\rangle\rangle \cong \langle\langle a, b, c\rangle\rangle$. Repeating the same calculation with a, b in place of c yields the result.

3.3. COROLLARY. *Let* $a, b, c \in \mathbf{Q}^\times$ *and let* $\mathbf{H} = \langle 1, -1\rangle$. *Then*

$$\langle\langle a, b, c\rangle\rangle \cong \begin{cases} \langle\langle 1, 1, 1\rangle\rangle & \text{if } a, b, c > 0 \\ 4\mathbf{H} & \text{otherwise} . \end{cases}$$

3.4. THEOREM. $I^3\mathbf{Q}$ *is torsion-free.*

Proof. Corollary 3.3 shows that the only nonzero 3-fold Pfister form in $I^3\mathbf{Q}$ is $\langle\langle 1, 1, 1\rangle\rangle$. Therefore $I^3\mathbf{Q} \cong \mathbf{Z}$ and $I^3\mathbf{Q}$ is torsion-free.

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