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Autor: Grigorieff, Serge / Richard, Denis

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CONTRIBUTION À L'ÉTUDE D'UNE CONJECTURE DE THÉORIE DES NOMBRES PAR LE CODAGE ZBV

par Serge GRIGORIEFF et Denis RICHARD

ABSTRACT. In [RJ] J. Robinson asked whether first order arithmetic over the set **N** of non negative integers is definable in terms of the successor function S and the coprimeness predicate $\perp$ (where $a \perp b$ iff a and b have no common prime divisor). It turns out that this question is equivalent to the following conjecture of number theory: Is there an integer k such that for every pair (x, y) of integers, the equality $x = y$ holds if and only if $x + i$ and $y + i$ have the same prime divisors for $0 \leqslant i \leqslant k$? This conjecture, due to A. Woods, is itself closely linked to some open questions proposed by P. Erdös (see [EP]). From the results in [RJ], [WA] and [RD], first order arithmetic is expressible in terms of the successor function S, the coprimeness predicate $\perp$ and anyone of the predicates of the following list:

$$x < y ; x + y = z ; x \mid y \, (x \text{ divides } y) ; x \times y = z ;$$
$$y = m^x \text{ (for any fixed } m \geqslant 2) .$$

This paper intends

1°) to present some number theoretical results which are pertinent tools to develop methods essentially relevant to mathematical logic;

2°) to give a survey of the history of arithmetical definability;

3°) to present some results about J. Robinson's question which unify all previously known ones;

4°) to add to the previous list new predicates such as RES(x, p) (which means that p is prime and x is a quadratic residue modulo p), POW(y, x) (which means that y is a power of x) and weak restrictions of addition, multiplication and division.

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