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Autor: Grigorieff, Serge / Richard, Denis
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CONTRIBUTION
À L'ÉTUDE D'UNE CONJECTURE DE THÉORIE DES NOMBRES
PAR LE CODAGE ZBV

par Serge GRIGORIEFF et Denis RICHARD

ABSTRACT. In [RJ] J. Robinson asked whether first order arithmetic over the set \mathbf{N} of non negative integers is definable in terms of the successor function S and the coprimeness predicate \perp (where $a \perp b$ iff a and b have no common prime divisor). It turns out that this question is equivalent to the following conjecture of number theory: Is there an integer k such that for every pair (x, y) of integers, the equality $x = y$ holds if and only if $x + i$ and $y + i$ have the same prime divisors for $0 \leq i \leq k$? This conjecture, due to A. Woods, is itself closely linked to some open questions proposed by P. Erdős (see [EP]). From the results in [RJ], [WA] and [RD], first order arithmetic is expressible in terms of the successor function S , the coprimeness predicate \perp and anyone of the predicates of the following list:

$$\begin{aligned} & x < y; x + y = z; x \mid y \text{ (} x \text{ divides } y\text{)}; x \times y = z; \\ & y = m^x \text{ (for any fixed } m \geq 2\text{).} \end{aligned}$$

This paper intends

- 1°) to present some number theoretical results which are pertinent tools to develop methods essentially relevant to mathematical logic;
- 2°) to give a survey of the history of arithmetical definability;
- 3°) to present some results about J. Robinson's question which unify all previously known ones;
- 4°) to add to the previous list new predicates such as $\text{RES}(x, p)$ (which means that p is prime and x is a quadratic residue modulo p), $\text{POW}(y, x)$ (which means that y is a power of x) and weak restrictions of addition, multiplication and division.