Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 35 (1989)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE FIXED POINT SET OF A FINITE GROUP ACTION ON A HOMOLOGY FOUR SPHERE

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Anhang: Appendix

DOI: https://doi.org/10.5169/seals-57368

two normal representations are equal. In the topological case, by the results of Cappel and Shaneson topological equivalence of matrices in dimension 4 implies linear equivalence, so the statement of Theorem 5.1 makes sense also for a group of homeomorphism.

The proof given can be adapted to this more general case provided that the followings are true:

1. the topological Atiyah-Singer signature formula holds,
2. a locally flat S^2 in Σ has a normal bundle,
3. the argument in case 3 works with Homeo (S^1) instead of $SO(2)$.

Assertion 1 is proved, in the case of the semi-free action, in [21], page 188; assertion 2 follows from the work of Freedman, see [10]; assertion 3 is proved using the retraction Homeo (S^1) into $SO(2)$ given by the Poincaré number, see [7].

Appendix

LEMMA. *The extensions:*

$$\begin{array}{ccccccccc} 0 & \to & C_2 & \to & \tilde{A}_5 & \to & A_5 & \to & 0 \\ & & & & \downarrow & & \downarrow & & \\ 0 & \to & C_2 & \to & A_5 \times A_5 & \to & A_5 \times A_5 & \to & 0 \\ & & & & \downarrow & & \downarrow {\scriptstyle (h,h')} & & \\ 0 & \to & C_2 & \to & SO(4) & \to & SO(3) \times SO(3) & \to & 0 \end{array}$$

are not split, h and h' can be any nontrivial representations of A_5 and f is either $(Id \times \{I\})$ or $(\{I\} \times Id)$.

Proof. Standard theory of group extensions and cohomology (see [4]) allows us to reduce to the:

PROPOSITION. *Any non trivial homomorphism $A_5 \xrightarrow{i} SO(3)$ induces an isomorphism $Z/2 = H^2(BSO(3); Z/2) \xrightarrow{i} H^2(BA_5; Z/2) = Z/2$.*

Proof of the Proposition. If the corresponding extension is split, then $Z/2 \times A_5 \subset S^3$, but $A_5 = 60$ so there exists a $Z/2 \subset A_5$ so $Z/2 \times Z/2$ would act freely on S^3, which cannot happen.