Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 35 (1989)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: REMARK ON MEROMORPHIC DIFFERENTIALS IN OPEN RIEMANN SURFACES

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Bibliographie

DOI: https://doi.org/10.5169/seals-57363

it by some meromorphic function with suitably chosen zeroes and poles in V and with "ones" of sufficiently large multiplicities at the points b_j. Such a function exists because a meromorphic function on V with zeroes and poles prescribed can be multiplied by the exponential of a suitable holomorphic function in order that the product has the desired "ones" with at least the desired multiplicities.

With the notation of the previous paragraph, we can already state the following.

THEOREM. *There exists a meromorphic differential in* V *with divisor* $\delta_0 - \delta$, *with precisely the singular parts defined at the* $\{b_j\}_{j \in \mathbf{N}}$ *by the* $P_j(1/z_j)\,dz_j$, *and with prescribed periods at the cycles of the canonical homology basis* F.

Proof. By applying an easy induction argument based on Lemma 2 to the sequence (U_n) we obtain a holomorphic function h in V with divisor $\geqslant \delta$ and such that $e^h\omega$ has the prescribed periods. Since e^h has at every b_j a "one" of multiplicity $\geqslant m_j (j \in \mathbf{N})$, we also deduce that $e^h\omega$ has the same singular parts that ω.

COROLLARY. *For a meromorphic function* f *in* V *it is possible to prescribe the divisors of* f *and* df, *provided that they are compatible (in the obvious sense), and the periods of* $d \log f$ *(being of course integral multiples of* $2\pi i$*) along curves defining any canonical homology basis of* V *(whenever these curves contain none of the zeroes or poles of* f*).*

Proof. If a meromorphic differential ω in V is chosen with only simple poles (corresponding to the zeroes and poles of f), suitable integral residues at these poles, suitable zeroes (corresponding to the zeroes of df at which f does not vanish) and the prescribed periods, it must be of the form $d \log f$, with f having all desired properties.

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(Reçu le 10 août 1988)

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