

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 35 (1989)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** THE CANTOR SET AND A GEOMETRIC CONSTRUCTION  
**Autor:** Pavone, Marco  
**Kapitel:** Introduction  
**DOI:** <https://doi.org/10.5169/seals-57361>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 27.04.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## THE CANTOR SET AND A GEOMETRIC CONSTRUCTION

by Marco PAVONE

### INTRODUCTION

The Cantor ternary set consists of all those real numbers  $x$  in  $[0, 1]$  which have a ternary expansion  $x = \sum_{n=1}^{\infty} a_n/3^n$  for which  $a_n$  is never 1. Equivalently,  $C$  can be obtained in a purely geometrical fashion by first removing from  $[0, 1]$  the middle third  $(1/3, 2/3)$ , then removing the middle thirds  $(1/9, 2/9)$  and  $(7/9, 8/9)$  of the remaining intervals, and so on ( $C$  will be exactly the complement of the countable union of the removed intervals). If  $x = \sum_{n=1}^{\infty} a_n/3^n$  is in  $C$ , the geometric interpretation of its ternary expansion is that  $x$  is the unique point in  $[0, 1]$  which is reached by first staying to the left or to the right of  $(1/3, 2/3)$  if  $a_1 = 0$  or  $a_1 = 2$  respectively, then staying to the left or to the right of the next removed interval if  $a_2 = 0$  or  $a_2 = 2$  respectively, and so on. It follows from the construction that  $C$  is a nowhere dense closed subset of  $[0, 1]$ .

A well known property of  $C$  is that any real number in  $[0, 2]$  can be written as the sum of two numbers in  $C$ . The purpose of this note is to give an elementary proof of  $C + C = [0, 2]$  which only uses the geometric definition of  $C$ . A refinement of the proof shows in fact that for any  $k$  in  $[0, 2]$  there exists either a finite or an uncountable number of pairs  $x, y$  from  $C$  such that  $x + y = k$ . We also discuss the analogy between this decomposition result and certain properties of continued fractions.

### THE GEOMETRIC CONSTRUCTION

We set, as usual,  $C \times C = \{(x, y) \in \mathbf{R}^2 : x, y \in C\}$ . Then  $C + C = [0, 2]$  can be geometrically restated as

(\*) for any  $k$  in  $[0, 2]$  the line  $x + y = k$  intersects  $C \times C$  in at least one point.