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where  $\equiv$  indicates that there is an orientation preserving diffeomorphism of pairs which is concordant to the identity map as a diffeomorphism of the ambient space M.

Our results suggest that I(M, L) and  $I_0(M, L)$  depend only on the order of a meridian of L in  $\pi_1(M-L)$  or  $H_1(M-L; \mathbb{Z})$ . Roughly speaking, according as the order is infinite, 1, or p(1 , they can be distinguishedby (at least) these three types:

*Type 1*  $I(M, L) = \{0N\},\$ 

Type 2  $I(M, L) = \mathscr{K}_n, \quad I_0(M, L) = \ker \sigma,$ 

 $Type \ 3 \quad \{0\} \underset{\neq}{\subset} I(M, L) \underset{\neq}{\subset} \mathscr{K}_n, \quad \{0\} \underset{\neq}{\subset} I_0(M, L) \underset{\neq}{\subset} \ker \sigma,$ 

(see section 4 for  $\sigma(S^{n+2}, K)$ ).

We refer the reader to 1.1, 2.6, 3.4, 5.1, 5.2, and 5.8 for the precise statement.

This paper consists of five sections. In Section 1, we deduce a necessary condition for  $I_0(M, L)$ , which is valid for any (M, L). We treat type 1 in Section 2. Type 2 is discussed in Sections 3, 4 and type 3 is discussed in Section 5. We will find that type 3 is closely related to the generalized Smith conjecture.

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§ 1. General remarks on  $I_0(M, L)$ 

It is known (and it is easily verified) that the signature of a Seifert surface of an oriented *n*-knot K in  $S^{n+2}$  is independent of the choice of a Seifert surface; so it is an invariant of the oriented knot K. The invariant is called the signature of the knot K and denoted by Sign  $(S^{n+2}, K)$ . We note that Sign  $(S^{n+2}, K)$  is trivially zero unless  $n + 1 \equiv 0$  (4).

As is seen in Section 3, there is a pair  $(M^{n+2}, L^n)$  such that  $I(M, L) = \mathscr{K}_n$  for any  $n \ge 3$ . In contrast, we can deduce a necessary condition for  $I_0(M, L)$  which holds for any pair (M, L).

THEOREM 1.1. If  $(S^{n+2}, K) \in I_0(M, L)$ , then  $Sign(S^{n+2}, K) = 0$ .

*Proof.* Let V be a Seifert surface of K. Since  $S^{n+2} = \partial D^{n+3}$ , we can push the interior of V into the interior of  $D^{n+3}$  so that V is transverse to  $S^{n+2}$ . This yields an oriented pair  $(D^{n+3}, V)$  having  $(S^{n+2}, K)$  as the boundary.

The boundary connected sum  $(M, L) \times I \nmid (D^{n+3}, V)$  gives a cobordism between  $(M, L) \notin (S^{n+2}, K)$  and (M, L). We note that the ambient space of the cobordism is diffeomorphic to  $M \times I$ . Since  $(S^{n+2}, K) \in I_0(M, L)$ , there is an orientation preserving diffeomorphism  $f: (M, L) \notin (S^{n+2}, K) \to (M, L)$  which is concordant to the identity when regarded as a diffeomorphism of the ambient space M. We paste togethor  $(M, L) # (S^{n+2}, K)$  and (M, L) by f to get an oriented pair of closed manifolds. Since f is concordant to the identity, the resulting ambient space is diffeomorphic to  $M \times S^1$ . We shall denote by X the resulting oriented closed submanifold of  $M \times S^1$ .

The additivity property of the signature (see [AS, p. 588]) says that

$$\operatorname{Sign} X = \operatorname{Sign} L \times I + \operatorname{Sign} V = \operatorname{Sign} V,$$

where Sign  $L \times I = 0$  follows easily from the definition of the signature of a manifold with boundary. By the Hirzebruch signature theorem (see  $[MS, \S 19])$  we have

$$\operatorname{Sign} X = \mathscr{L}(X)[X]$$

where the right hand side means the Hirzebruch L-class  $\mathscr{L}(X)$  of X evaluated on the fundamental class [X] of X. In the sequel we shall show  $\mathscr{L}(X)[X] = 0.$ 

Let  $j: X \to M \times S^1$  be the inclusion map. Then it is not difficult to see that

(1.2) 
$$j_*[X] = [L \times S^1]$$
 in  $H_{n+1}(M \times S^1; \mathbb{Z})$ 

where  $[L \times S^1]$  denotes the homology class represented by  $L \times S^1$ .

Let v be the normal bundle to X in  $M \times S^1$ . By the multiplicativity of *L*-class we have

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$$\mathscr{L}(X) = \mathscr{L}(v)^{-1} j^* \mathscr{L}(M \times S^1)$$

$$\mathscr{L}(M \times S^1) = \mathscr{L}(M) \times \mathscr{L}(S^1) = \pi^* \mathscr{L}(M)$$

where  $\pi: M \times S^1 \to M$  is the projection map. Since dim v = 2, we have

(1.4) 
$$\mathscr{L}(v) = 1 + p_1(v)/3 = 1 + e(v)^2/3$$

where  $p_1$  and e denote the first Pontrjagin class and the Euler class respectively.

On the other hand it is known that

(0)

(1.5) 
$$e(v) = j^* j_1(1)$$

where  $j_1 : H^q(X; \mathbb{Z}) \to H^{q+2}(M \times S^1; \mathbb{Z})$  denotes the Gysin homomorphism and  $1 \in H^0(X; \mathbb{Z})$  is the unit element. Remember the definition of  $j_1$ . It is defined so that the following diagram commutes:

$$\begin{array}{cccc} H^{q}(X\,;\,\mathbf{Z}) & \stackrel{J_{!}}{\to} & H^{q+2}(M\times S^{1}\,;\,\mathbf{Z}) \\ & \downarrow & & \uparrow & & \uparrow & \\ & & \downarrow & & \uparrow & \\ H_{n+1-q}(X\,;\,\mathbf{Z}) & \stackrel{j_{*}}{\to} & H_{n+1-q}(M\times S^{1}\,;\,\mathbf{Z}) \end{array}$$

where the vertical maps are the Poincaré dualities. It says that

 $j_!(1) \cap [M \times S^1] = j_*[X].$ 

This together with (1.2) means that

$$j_{1}(1) \in \pi^{*}H^{2}(M; \mathbb{Z})$$
.

Hence it follows from (1.4) and (1.5) that

 $\mathscr{L}(\mathbf{v}) \in j^*\pi^*H^*(M; Q)$ 

and hence

 $\mathscr{L}(X) \in j^* \pi^* H^*(M; Q)$ 

by (1.3). This together with (1.2) implies that

 $\mathscr{L}(X)[X] = 0. \qquad \text{Q.E.D.}$ 

Theorem 1.1 gives a necessary condition for  $(S^{n+2}, K)$  to belong to  $I_0(M, L)$ . When we consider the converse problem, i.e. the problem to find  $(S^{n+2}, K)$  in  $I_0(M, L)$ , we apply the relative s-cobordism theorem. We shall state it as a lemma for later convenience's sake.

LEMMA 1.6. Suppose there exists a cobordism (U, Z) between (M, L)#  $(S^{n+2}, K)$  and (M, L) such that

(1) Z is diffeomorphic to  $L \times I$ ,

(2) the exterior E(Z) of Z is an s-cobordism relative boundary. Then  $(S^{n+2}, K) \in I_0(M, L)$ .

*Proof.* The relative s-cobordism theorem says that E(Z) is diffeomorphic to  $E(L) \times I$  where the diffeomorphism can be taken as the identity on  $E(L) \times \{0\}$  and  $(\partial E(L)) \times I$ . Therefore it extends to a diffeomorphism:  $(U, Z) \rightarrow (M, L) \times I$  which is the identity on the 0-level. This means that  $(S^{n+2}, K) \in I_0(M, L)$ . Q.E.D.