Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 35 (1989)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: KNOTTING CODIMENSION 2 SUBMANIFOLDS LOCALLY
Autor: Masuda, Mikiya / Sakuma, Makoto
Kapitel: §1. General remarks on $I_0(M,L)$
DOI: https://doi.org/10.5169/seals-57360

Nutzungsbedingungen
Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Mehr erfahren

Conditions d'utilisation
L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. En savoir plus

Terms of use
The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. Find out more

Download PDF: 19.01.2026

ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch

where $\equiv$ indicates that there is an orientation preserving diffeomorphism of pairs which is concordant to the identity map as a diffeomorphism of the ambient space M.

Our results suggest that $I(M, L)$ and $I_0(M, L)$ depend only on the order of a meridian of L in $\pi_1(M-L)$ or $H_1(M-L; \mathbf{Z})$. Roughly speaking, according as the order is infinite, 1, or p $(1<p<\infty)$, they can be distinguished by (at least) these three types:

Type 1 $\quad I(M, L) = \{0N\}$,

Type 2 $\quad I(M, L) = \mathscr{K}_n, \quad I_0(M, L) = \ker\sigma$,

Type 3 $\quad \{0\} \subsetneqq I(M, L) \subsetneqq \mathscr{K}_n, \quad \{0\} \subsetneqq I_0(M, L) \subsetneqq \ker\sigma$,

(see section 4 for $\sigma(S^{n+2}, K)$).

We refer the reader to 1.1, 2.6, 3.4, 5.1, 5.2, and 5.8 for the precise statement.

This paper consists of five sections. In Section 1, we deduce a necessary condition for $I_0(M, L)$, which is valid for any (M, L). We treat type 1 in Section 2. Type 2 is discussed in Sections 3, 4 and type 3 is discussed in Section 5. We will find that type 3 is closely related to the generalized Smith conjecture.

The authors would like to express their hearty thanks to Professors A. Kawauchi and T. Maeda for helpful conversations and suggestions.

§ 1. General remarks on $I_0(M, L)$

It is known (and it is easily verified) that the signature of a Seifert surface of an oriented n-knot K in S^{n+2} is independent of the choice of a Seifert surface; so it is an invariant of the oriented knot K. The invariant is called the signature of the knot K and denoted by $\mathrm{Sign}(S^{n+2}, K)$. We note that $\mathrm{Sign}(S^{n+2}, K)$ is trivially zero unless $n+1 \equiv 0$ (4).

As is seen in Section 3, there is a pair (M^{n+2}, L^n) such that $I(M, L) = \mathscr{K}_n$ for any $n \geqslant 3$. In contrast, we can deduce a necessary condition for $I_0(M, L)$ which holds for any pair (M, L).

THEOREM 1.1. *If* $(S^{n+2}, K) \in I_0(M, L)$, *then* $\mathrm{Sign}(S^{n+2}, K) = 0$.

Proof. Let V be a Seifert surface of K. Since $S^{n+2} = \partial D^{n+3}$, we can push the interior of V into the interior of D^{n+3} so that V is transverse to S^{n+2}. This yields an oriented pair (D^{n+3}, V) having (S^{n+2}, K) as the boundary.

The boundary connected sum $(M, L) \times I \natural (D^{n+3}, V)$ gives a cobordism between $(M, L) \# (S^{n+2}, K)$ and (M, L). We note that the ambient space of the cobordism is diffeomorphic to $M \times I$. Since $(S^{n+2}, K) \in I_0(M, L)$, there is an orientation preserving diffeomorphism $f: (M, L) \# (S^{n+2}, K) \to (M, L)$ which is concordant to the identity when regarded as a diffeomorphism of the ambient space M. We paste togethor $(M, L) \# (S^{n+2}, K)$ and (M, L) by f to get an oriented pair of closed manifolds. Since f is concordant to the identity, the resulting ambient space is diffeomorphic to $M \times S^1$. We shall denote by X the resulting oriented closed submanifold of $M \times S^1$.

The additivity property of the signature (see [AS, p. 588]) says that

$$\operatorname{Sign} X = \operatorname{Sign} L \times I + \operatorname{Sign} V = \operatorname{Sign} V,$$

where $\operatorname{Sign} L \times I = 0$ follows easily from the definition of the signature of a manifold with boundary. By the Hirzebruch signature theorem (see [MS, § 19]) we have

$$\operatorname{Sign} X = \mathscr{L}(X)[X]$$

where the right hand side means the Hirzebruch L-class $\mathscr{L}(X)$ of X evaluated on the fundamental class $[X]$ of X. In the sequel we shall show $\mathscr{L}(X)[X] = 0$.

Let $j: X \to M \times S^1$ be the inclusion map. Then it is not difficult to see that

(1.2) $$j_*[X] = [L \times S^1] \quad \text{in} \quad H_{n+1}(M \times S^1; \mathbf{Z})$$

where $[L \times S^1]$ denotes the homology class represented by $L \times S^1$.

Let ν be the normal bundle to X in $M \times S^1$. By the multiplicativity of L-class we have

(1.3) $$\begin{aligned} \mathscr{L}(X) &= \mathscr{L}(\nu)^{-1} j^* \mathscr{L}(M \times S^1) \\ \mathscr{L}(M \times S^1) &= \mathscr{L}(M) \times \mathscr{L}(S^1) = \pi^* \mathscr{L}(M) \end{aligned}$$

where $\pi: M \times S^1 \to M$ is the projection map. Since $\dim \nu = 2$, we have

(1.4) $$\mathscr{L}(\nu) = 1 + p_1(\nu)/3 = 1 + e(\nu)^2/3$$

where p_1 and e denote the first Pontrjagin class and the Euler class respectively.

On the other hand it is known that

(1.5) $$e(\nu) = j^* j_!(1)$$

where $j_! : H^q(X;\mathbf{Z}) \to H^{q+2}(M\times S^1;\mathbf{Z})$ denotes the Gysin homomorphism and $1 \in H^0(X;\mathbf{Z})$ is the unit element. Remember the definition of $j_!$. It is defined so that the following diagram commutes:

$$\begin{array}{ccc} H^q(X;\mathbf{Z}) & \xrightarrow{j_!} & H^{q+2}(M\times S^1;\mathbf{Z}) \\ \downarrow {\scriptstyle \cap[X]} & & \downarrow {\scriptstyle \cap[M\times S^1]} \\ H_{n+1-q}(X;\mathbf{Z}) & \xrightarrow{j_*} & H_{n+1-q}(M\times S^1;\mathbf{Z}) \end{array}$$

where the vertical maps are the Poincaré dualities. It says that

$$j_!(1) \cap [M\times S^1] = j_*[X].$$

This together with (1.2) means that

$$j_!(1) \in \pi^* H^2(M;\mathbf{Z}).$$

Hence it follows from (1.4) and (1.5) that

$$\mathscr{L}(\nu) \in j^*\pi^*H^*(M;Q)$$

and hence

$$\mathscr{L}(X) \in j^*\pi^*H^*(M;Q)$$

by (1.3). This together with (1.2) implies that

$$\mathscr{L}(X)[X] = 0. \qquad \text{Q.E.D.}$$

Theorem 1.1 gives a necessary condition for (S^{n+2}, K) to belong to $I_0(M, L)$. When we consider the converse problem, i.e. the problem to find (S^{n+2}, K) in $I_0(M, L)$, we apply the relative s-cobordism theorem. We shall state it as a lemma for later convenience's sake.

LEMMA 1.6. *Suppose there exists a cobordism* (U, Z) *between* $(M, L) \# (S^{n+2}, K)$ *and* (M, L) *such that*

(1) Z *is diffeomorphic to* $L\times I$,

(2) *the exterior* $E(Z)$ *of* Z *is an s-cobordism relative boundary.*

Then $(S^{n+2}, K) \in I_0(M, L)$.

Proof. The relative s-cobordism theorem says that $E(Z)$ is diffeomorphic to $E(L)\times I$ where the diffeomorphism can be taken as the identity on $E(L)\times\{0\}$ and $(\partial E(L))\times I$. Therefore it extends to a diffeomorphism: $(U, Z) \to (M, L)\times I$ which is the identity on the 0-level. This means that $(S^{n+2}, K) \in I_0(M, L)$. Q.E.D.