

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 35 (1989)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: REPRESENTATION OF EVERY RATIONAL NUMBER AS AN ALGEBRAIC SUM OF FIFTH POWERS OF RATIONAL NUMBERS
Autor: Choudhry, Ajai
DOI: <https://doi.org/10.5169/seals-57359>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 27.04.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

REPRESENTATION
OF EVERY RATIONAL NUMBER AS AN ALGEBRAIC SUM
OF FIFTH POWERS OF RATIONAL NUMBERS

by Ajai CHOUDHRY

The representation of a rational number as an algebraic sum of k^{th} powers of rational numbers has been considered by Subba Rao [1]. Let $g_1(k)$ be defined as the least integer s such that every rational number r can be expressed in the form

$$(1) \quad r = a_1x_1^k + a_2x_2^k + \dots + a_sx_s^k$$

where $a_i = \pm 1$ and all of the values of x_i are rational. It has been shown [1] that

$$g_1(5) \leq 8.$$

In this note we shall prove that $g_1(5) \leq 6$. We shall also obtain a parametric solution of the Diophantine equation

$$(2) \quad x_1^5 + x_2^5 + \dots + x_s^5 = 0 \quad \text{for} \quad s \geq 6.$$

The result follows from the identity

$$(3) \quad \begin{aligned} & (t^6x - t^{15} - t^5 - 1)^5 + (t^3x + t^2)^5 + (x - t^{14})^5 \\ & - (t^6x - t^{15} - t^5 + 1)^5 - (t^3x - 2t^{12} - t^2)^5 - (x + t^{14})^5 \\ & = -2(t^{40} + t^{30} - t^{10} - 1)(20t^{11}x + t^{30} - 12t^{20} - 8t^{10} - 1). \end{aligned}$$

As the expression on the right-hand side of (3) is linear in x , it can be equated to any rational number r , and solved for x , which leads to the result

$$g_1(5) \leq 6.$$

Thus, every rational number can be expressed as the algebraic sum of at most six fifth powers of rational numbers, and since in the above discussion, t can be taken as any rational number except that $t \neq 0, \pm 1$, this can be done in infinitely many ways.

To solve (2) for $s = 6$, we equate the right-hand side of (3) to 0, and obtain the result

$$\begin{aligned} & (t^{36} + 8t^{26} + 12t^{16} + 20t^{11} - t^6)^5 + (t^{33} - 12t^{23} - 28t^{13} - t^3)^5 \\ & + (t^{30} + 20t^{25} - 12t^{20} - 8t^{10} - 1)^5 - (t^{36} + 8t^{26} + 12t^{16} - 20t^{11} - t^6)^5 \\ & - (t^{33} + 28t^{23} + 12t^{13} - t^3)^5 - (t^{30} - 20t^{25} - 12t^{20} - 8t^{10} - 1)^5 = 0. \end{aligned}$$

This result has earlier been given by Moessner [2].

To solve (2) for $s = 6 + m$, $m > 0$, we simply equate the right hand side of (3) to $\sum_{i=1}^m x_i^5$ where x_i are any rational numbers, and solve for x , which leads to a solution of (2) for $s > 6$. Solutions in integers can be obtained by multiplying by a suitable constant.

REFERENCES

- [1] SUBBA RAO, K. Representation of Every Number as a Sum of Rational k^{th} Powers. *Journal London Math. Soc.* 13 (1938), 14-16.
 [2] MOESSNER, A. Due Sistemi Diofantei. *Boll. Un. Mat. Ital. Ser. 3*, 6 (1951), 117-118.

(Reçu le 29 janvier 1988)

Ajai Choudhry

Embassy of India
 Rejtana 15
 Warsaw (Poland)