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REPRESENTATION
OF EVERY RATIONAL NUMBER AS AN ALGEBRAIC SUM
OF FIFTH POWERS OF RATIONAL NUMBERS

by Ajai CHOUDHRY

The representation of a rational number as an algebraic sum of k^{th} powers of rational numbers has been considered by Subba Rao [1]. Let $g_1(k)$ be defined as the least integer s such that every rational number r can be expressed in the form

$$(1) \quad r = a_1 x_1^k + a_2 x_2^k + \dots + a_s x_s^k$$

where $a_i = \pm 1$ and all of the values of x_i are rational. It has been shown [1] that

$$g_1(5) \leq 8.$$

In this note we shall prove that $g_1(5) \leq 6$. We shall also obtain a parametric solution of the Diophantine equation

$$(2) \quad x_1^5 + x_2^5 + \dots + x_s^5 = 0 \quad \text{for} \quad s \geq 6.$$

The result follows from the identity

$$(3) \quad \begin{aligned} & (t^6 x - t^{15} - t^5 - 1)^5 + (t^3 x + t^2)^5 + (x - t^{14})^5 \\ & - (t^6 x - t^{15} - t^5 + 1)^5 - (t^3 x - 2t^{12} - t^2)^5 - (x + t^{14})^5 \\ & = - 2(t^{40} + t^{30} - t^{10} - 1)(20t^{11}x + t^{30} - 12t^{20} - 8t^{10} - 1). \end{aligned}$$

As the expression on the right-hand side of (3) is linear in x , it can be equated to any rational number r , and solved for x , which leads to the result

$$g_1(5) \leq 6.$$

Thus, every rational number can be expressed as the algebraic sum of at most six fifth powers of rational numbers, and since in the above discussion, t can be taken as any rational number except that $t \neq 0, \pm 1$, this can be done in infinitely many ways.

To solve (2) for $s = 6$, we equate the right-hand side of (3) to 0, and obtain the result

$$\begin{aligned} & (t^{36} + 8t^{26} + 12t^{16} + 20t^{11} - t^6)^5 + (t^{33} - 12t^{23} - 28t^{13} - t^3)^5 \\ & + (t^{30} + 20t^{25} - 12t^{20} - 8t^{10} - 1)^5 - (t^{36} + 8t^{26} + 12t^{16} - 20t^{11} - t^6)^5 \\ & - (t^{33} + 28t^{23} + 12t^{13} - t^3)^5 - (t^{30} - 20t^{25} - 12t^{20} - 8t^{10} - 1)^5 = 0. \end{aligned}$$

This result has earlier been given by Moessner [2].

To solve (2) for $s = 6 + m$, $m > 0$, we simply equate the right hand side of (3) to $\sum_{i=1}^m x_i^5$ where x_i are any rational numbers, and solve for x , which leads to a solution of (2) for $s > 6$. Solutions in integers can be obtained by multiplying by a suitable constant.

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