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REPRESENTATION  
OF EVERY RATIONAL NUMBER AS AN ALGEBRAIC SUM  
OF FIFTH POWERS OF RATIONAL NUMBERS

by Ajai CHOUDHRY

The representation of a rational number as an algebraic sum of  $k^{\text{th}}$  powers of rational numbers has been considered by Subba Rao [1]. Let  $g_1(k)$  be defined as the least integer  $s$  such that every rational number  $r$  can be expressed in the form

$$(1) \quad r = a_1x_1^k + a_2x_2^k + \dots + a_sx_s^k$$

where  $a_i = \pm 1$  and all of the values of  $x_i$  are rational. It has been shown [1] that

$$g_1(5) \leq 8.$$

In this note we shall prove that  $g_1(5) \leq 6$ . We shall also obtain a parametric solution of the Diophantine equation

$$(2) \quad x_1^5 + x_2^5 + \dots + x_s^5 = 0 \quad \text{for} \quad s \geq 6.$$

The result follows from the identity

$$(3) \quad \begin{aligned} & (t^6x - t^{15} - t^5 - 1)^5 + (t^3x + t^2)^5 + (x - t^{14})^5 \\ & - (t^6x - t^{15} - t^5 + 1)^5 - (t^3x - 2t^{12} - t^2)^5 - (x + t^{14})^5 \\ & = -2(t^{40} + t^{30} - t^{10} - 1)(20t^{11}x + t^{30} - 12t^{20} - 8t^{10} - 1). \end{aligned}$$

As the expression on the right-hand side of (3) is linear in  $x$ , it can be equated to any rational number  $r$ , and solved for  $x$ , which leads to the result

$$g_1(5) \leq 6.$$

Thus, every rational number can be expressed as the algebraic sum of at most six fifth powers of rational numbers, and since in the above discussion,  $t$  can be taken as any rational number except that  $t \neq 0, \pm 1$ , this can be done in infinitely many ways.

To solve (2) for  $s = 6$ , we equate the right-hand side of (3) to 0, and obtain the result

$$\begin{aligned} & (t^{36} + 8t^{26} + 12t^{16} + 20t^{11} - t^6)^5 + (t^{33} - 12t^{23} - 28t^{13} - t^3)^5 \\ & + (t^{30} + 20t^{25} - 12t^{20} - 8t^{10} - 1)^5 - (t^{36} + 8t^{26} + 12t^{16} - 20t^{11} - t^6)^5 \\ & - (t^{33} + 28t^{23} + 12t^{13} - t^3)^5 - (t^{30} - 20t^{25} - 12t^{20} - 8t^{10} - 1)^5 = 0. \end{aligned}$$

This result has earlier been given by Moessner [2].

To solve (2) for  $s = 6 + m$ ,  $m > 0$ , we simply equate the right hand side of (3) to  $\sum_{i=1}^m x_i^5$  where  $x_i$  are any rational numbers, and solve for  $x$ , which leads to a solution of (2) for  $s > 6$ . Solutions in integers can be obtained by multiplying by a suitable constant.

#### REFERENCES

- [1] SUBBA RAO, K. Representation of Every Number as a Sum of Rational  $k^{\text{th}}$  Powers. *Journal London Math. Soc.* 13 (1938), 14-16.  
 [2] MOESSNER, A. Due Sistemi Diofantei. *Boll. Un. Mat. Ital. Ser. 3*, 6 (1951), 117-118.

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