

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 35 (1989)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON SUMS OF FOURIER COEFFICIENTS OF CUSP FORMS
Autor: Hafner, James Lee / Ivi, Aleksandar

Kurzfassung

DOI: <https://doi.org/10.5169/seals-57381>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 29.04.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

ON SUMS OF FOURIER COEFFICIENTS OF CUSP FORMS

by James Lee HAFNER and Aleksandar IVIĆ

ABSTRACT. In this paper, we provide both upper and lower estimates for the summatory functions of coefficients of cusp forms. Some of the results also hold for Maass wave forms. The proofs are essentially applications of general results of Chandrasekharan and Narasimhan and of Redmond together with some non-trivial results of Murty, Rankin and others.

The purpose of this note is to collect some hitherto unnoticed or unpublished results concerning the summatory functions of the Fourier coefficients of cusp forms, including Maass wave forms. Most of the results are obtained by direct application of some general theorems about Dirichlet series satisfying a functional equation and more specific (and deeper) results concerning cusp form coefficients. One of our purposes in writing this paper is to get some of the "folklore" into print.

As usual, we will need some notation. Let $F(z)$ be a cusp form of weight k for $\Gamma = PSL(2, Z)$. (Our results also hold for cusp forms on congruence subgroups but this extra generality would only make the notation more complicated.) Write $F(z)$ in a Fourier series:

$$F(z) = \sum_{n=1}^{\infty} a(n)e^{2\pi inz}. \quad (\text{Im } z > 0)$$

If we assume that F is a normalized eigenfunction for the Hecke operators, then we have that the $a(n)$'s are multiplicative, real valued, and satisfy

$$|a(n)| \leq d(n)n^{(k-1)/2},$$

and

$$(1) \quad \sum_{n \leq x} a(n)^2 = Ax^k + O(x^{k-2/5}).$$