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is commutative and as both  $\delta$ 's are isomorphisms it suffices to show that

$$\delta^{-1} \circ c_1: \text{Hom}(G, \mathbf{C}^*) \rightarrow H^1(G, \mathbf{Q}/\mathbf{Z}) \simeq \text{Hom}(G, \mathbf{Q}/\mathbf{Z})$$

is an isomorphism.

But by inspection  $\delta^{-1} \circ c_1(\varphi) = \delta^{-1}(u) \circ \varphi$ , and as  $u$  is a  $\widehat{\mathbf{Z}} = \text{End}_{\mathbf{Z}}(\mu_\infty)$  generator for  $H^2(\mu_\infty, \mathbf{Z}) \simeq \widehat{\mathbf{Z}}$ ,  $\delta^{-1}(u)$  is an isomorphism

$$\delta^{-1}(u): \mu_\infty \xrightarrow{\sim} \mathbf{Q}/\mathbf{Z}.$$

#### SECTION 4. CHERN CLASSES FOR LOCALLY FINITE GROUPS

The definition will be based on the following two observations. In the following, let  $G = \varinjlim G_k$  be a locally finite group where  $\{G_k\}$  is a family of finite subgroups.

LEMMA. *Let*

$$\varphi: G \rightarrow \text{Gl}_n(\mathbf{C})$$

*be a representation of  $G$ . Then  $\varphi$  is uniquely determined by its restrictions*

$$\varphi_k: G_k \rightarrow \text{Gl}_n(\mathbf{C}).$$

*Conversely given a family of compatible representations  $\varphi_k: G_k \rightarrow \text{Gl}_n(\mathbf{C})$ , there exists a unique  $\varphi: G \rightarrow \text{Gl}_n(\mathbf{C})$  which restricts to  $\varphi_k$  for all  $k$ .*

*Proof.* From the universal property of the direct limit, we have

$$\text{Hom}(G, \text{Gl}_n(\mathbf{C})) \cong \varprojlim \text{Hom}(G_k, \text{Gl}_n(\mathbf{C})).$$

PROPOSITION. *For all  $i \geq 0$ , the natural map*

$$H^i(G, \mathbf{Z}) \cong \varprojlim H^i(G_k, \mathbf{Z}).$$

*is an isomorphism.*

*Proof.* Obvious for  $i = 0, 1$ . For  $i \geq 1$ , the homology groups

$$H_i(G, \mathbf{Z}) = \varinjlim H_i(G_k, \mathbf{Z})$$

are all abelian torsion groups.

Now by the universal coefficient theorem ( $i \geq 1$ )

$$H^i(G, \mathbf{Z}) \cong \text{Ext}_{\mathbf{Z}}^1(H_{i-1}(G, \mathbf{Z}), \mathbf{Z})$$

and as observed above, for  $i \geq 2$ ,  $H_{i-1}(G, \mathbf{Z})$  is torsion. Thus

$$\begin{aligned} \text{Ext}_{\mathbf{Z}}^1(H_{i-1}(G, \mathbf{Z}), \mathbf{Z}) &\cong \text{Hom}_{\mathbf{Z}}(H_{i-1}(G, \mathbf{Z}), \mathbf{Q}/\mathbf{Z}) \\ &\cong \text{Hom}_{\mathbf{Z}}(\lim_{\rightarrow} H_{i-1}(G_k, \mathbf{Z}), \mathbf{Q}/\mathbf{Z}) \cong \lim_{\leftarrow} \text{Hom}_{\mathbf{Z}}(H_{i-1}(G_k, \mathbf{Z}), \mathbf{Q}/\mathbf{Z}) \\ &\simeq \lim_{\leftarrow} \text{Ext}_{\mathbf{Z}}^1(H_{i-1}(G_k, \mathbf{Z}), \mathbf{Z}) \cong \lim_{\leftarrow} H^i(G_k, \mathbf{Z}). \end{aligned}$$

Combining these two results, there exists for a representation

$$\phi: G \rightarrow Gl_n(\mathbf{C})$$

of a locally finite group  $G = \lim_{\rightarrow} G_k$  a unique cohomology class  $c \cdot (\phi) \in H^{**}(G, \mathbf{Z})$  such that for all  $k$

$$\text{res}_{G_k}^G(c \cdot (\phi)) = c \cdot (\phi_k)$$

Using this uniqueness result, it is easy to see that these classes satisfy the properties CH1, CH2 and CH3 and that they are uniquely determined by these properties.

## REFERENCES

- [1] ATIYAH, M. F. Characters and cohomology of finite groups. *Inst. Hautes Etudes Sci. Publ. Math.* 9 (1961), 2-64.
- [2] EVENS, L. On the Chern classes of representations of finite groups. *Trans. Amer. Math. Soc.* 115 (1965), 180-193.
- [3] EVENS, L. and D. KHAN. An integral Riemann-Roch formula for induced representations of finite groups. *Trans. Amer. Math. Soc.* 245 (1978), 309-330.
- [4] GREEN, J. A. The characters of the finite general linear groups. *Trans. Amer. Math. Soc.* 80 (1955), 402-447.
- [5] GROTHENDIECK, A. Classes de Chern des représentations linéaires des groupes discrets. In *Dix Exposés sur la Cohomologie étale des schémas*, North Holland, 1968.
- [6] KROLL, O. The cohomology of the finite general linear group. To appear in *J. Pure and Appl. Algebra*.
- [7] —— An algebraic characterisation of Chern classes of finite group representations. *Bull. London Math. Soc.* 19 (1987), 245-248.