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is commutative and as both  $\delta$ 's are isomorphisms it suffices to show that

$$\delta^{-1} \circ c_1: \text{Hom}(G, \mathbf{C}^*) \rightarrow H^1(G, \mathbf{Q}/\mathbf{Z}) \simeq \text{Hom}(G, \mathbf{Q}/\mathbf{Z})$$

is an isomorphism.

But by inspection  $\delta^{-1} \circ c_1(\varphi) = \delta^{-1}(u) \circ \varphi$ , and as  $u$  is a  $\hat{\mathbf{Z}} = \text{End}_{\mathbf{Z}}(\mu_{\infty})$  generator for  $H^2(\mu_{\infty}, \mathbf{Z}) \simeq \hat{\mathbf{Z}}$ ,  $\delta^{-1}(u)$  is an isomorphism

$$\delta^{-1}(u): \mu_{\infty} \xrightarrow{\sim} \mathbf{Q}/\mathbf{Z}.$$

#### SECTION 4. CHERN CLASSES FOR LOCALLY FINITE GROUPS

The definition will be based on the following two observations. In the following, let  $G = \varinjlim G_k$  be a locally finite group where  $\{G_k\}$  is a family of finite subgroups.

LEMMA. *Let*

$$\varphi: G \rightarrow \text{Gl}_n(\mathbf{C})$$

*be a representation of  $G$ . Then  $\varphi$  is uniquely determined by its restrictions*

$$\varphi_k: G_k \rightarrow \text{Gl}_n(\mathbf{C}).$$

*Conversely given a family of compatible representations  $\varphi_k: G_k \rightarrow \text{Gl}_n(\mathbf{C})$ , there exists a unique  $\varphi: G \rightarrow \text{Gl}_n(\mathbf{C})$  which restricts to  $\varphi_k$  for all  $k$ .*

*Proof.* From the universal property of the direct limit, we have

$$\text{Hom}(G, \text{Gl}_n(\mathbf{C})) \cong \varprojlim \text{Hom}(G_k, \text{Gl}_n(\mathbf{C})).$$

PROPOSITION. *For all  $i \geq 0$ , the natural map*

$$H^i(G, \mathbf{Z}) \cong \varprojlim H^i(G_k, \mathbf{Z}).$$

*is an isomorphism.*

*Proof.* Obvious for  $i = 0, 1$ . For  $i \geq 1$ , the homology groups

$$H_i(G, \mathbf{Z}) = \varinjlim H_i(G_k, \mathbf{Z})$$

are all abelian torsion groups.

Now by the universal coefficient theorem ( $i \geq 1$ )

$$H^i(G, \mathbf{Z}) \cong \text{Ext}_{\mathbf{Z}}^1(H_{i-1}(G, \mathbf{Z}), \mathbf{Z})$$

and as observed above, for  $i \geq 2$ ,  $H_{i-1}(G, \mathbf{Z})$  is torsion. Thus

$$\begin{aligned} \text{Ext}_{\mathbf{Z}}^1(H_{i-1}(G, \mathbf{Z}), \mathbf{Z}) &\cong \text{Hom}_{\mathbf{Z}}(H_{i-1}(G, \mathbf{Z}), \mathbf{Q}/\mathbf{Z}) \\ &\cong \text{Hom}_{\mathbf{Z}}(\varinjlim H_{i-1}(G_k, \mathbf{Z}), \mathbf{Q}/\mathbf{Z}) \cong \varprojlim \text{Hom}_{\mathbf{Z}}(H_{i-1}(G_k, \mathbf{Z}), \mathbf{Q}/\mathbf{Z}) \\ &\simeq \varprojlim \text{Ext}_{\mathbf{Z}}^1(H_{i-1}(G_k, \mathbf{Z}), \mathbf{Z}) \cong \varprojlim H^i(G_k, \mathbf{Z}). \end{aligned}$$

Combining these two results, there exists for a representation

$$\varphi: G \rightarrow \text{Gl}_n(\mathbf{C})$$

of a locally finite group  $G = \varinjlim G_k$  a unique cohomology class  $c.(\varphi) \in H^{**}(G, \mathbf{Z})$  such that for all  $k$

$$\text{res}_{G_k}^G(c.(\varphi)) = c.(\varphi_k)$$

Using this uniqueness result, it is easy to see that these classes satisfy the properties CH1, CH2 and CH3 and that they are uniquely determined by these properties.

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