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The validity of this hypothesis is the most interesting open problem about the **TH**. It might be found by checking a suitable guess about the minimal length for $\mathfrak{P}2$ with four pegs against the recursive solution which can easily be constructed using the fact, proved as Lemma 1, that the largest disc will not move more than three times.

In contrast to this recursive solution, the use of the hypothesis leads to a very elegant iterative solution to $\mathfrak{P}0$ with four (or more) pegs (see Hinz [26]), resembling algorithm i in 1.2.1, with the astonishing result that the transfer of 64 discs can be carried out in less than 6 hours (compare the time needed with three pegs, indicated in the Introduction!).

To conclude, it can be said that the invention of Edouard Lucas, besides its appeal as a puzzle for human beings as well as for computer performance, has been endowed with enough structure to be treated mathematically (the problem $\mathfrak{P}5 :=$ irregular $\rightarrow$ irregular without the "devine rule" (0) seems to have almost no mathematical structure), but not with so much to be trivial and uncapable of meaningful generalizations. As long as there are still open problems, a mathematical subject is not dead. The brahmins are alive and as long as they are still moving golden discs, the world will, according to legend, not fall to dust. Let us hope so!

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