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The validity of this hypothesis is the most interesting open problem about the **TH**. It might be found by checking a suitable guess about the minimal length for $\mathfrak{P}2$ with four pegs against the recursive solution which can easily be constructed using the fact, proved as Lemma 1, that the largest disc will not move more than three times.

In contrast to this recursive solution, the use of the hypothesis leads to a very elegant iterative solution to $\mathfrak{P}0$ with four (or more) pegs (see Hinz [26]), resembling algorithm i in 1.2.1, with the astonishing result that the transfer of 64 discs can be carried out in less than 6 hours (compare the time needed with three pegs, indicated in the Introduction!).

To conclude, it can be said that the invention of Edouard Lucas, besides its appeal as a puzzle for human beings as well as for computer performance, has been endowed with enough structure to be treated mathematically (the problem $\mathfrak{P}5 :=$ irregular $\rightarrow$ irregular without the "devine rule" (0) seems to have almost no mathematical structure), but not with so much to be trivial and uncapable of meaningful generalizations. As long as there are still open problems, a mathematical subject is not dead. The brahmins are alive and as long as they are still moving golden discs, the world will, according to legend, not fall to dust. Let us hope so!

REFERENCES

[1] AFRIAT, S. N. *The Ring of Linked Rings.* Duckworth (London), 1982.

[2] ALLARDICE, R. E. and A. Y. FRASER. La Tour d'Hanoï. *Proc. Edinburgh Math. Soc. 2* (1883-84), 50-53.

[3] BALL, W. W. R. *Mathematical recreations and problems of past and present times.* Macmillan (London), 1892 (2nd ed.).

[4] BENDISCH, J. Generalized Sequencing Problem "Towers of Hanoi". *Z. Oper. Res. 29* (1985), 31-45.

[5] BERMAN, G. and K. D. FRYER. *Introduction to combinatorics.* Academic Press (New York), 1972.

[6] BROUSSEAU, A. Tower of Hanoi with more pegs. *J. Recreational Math. 8* (1975-76), 169-178.

[7] CHARNIAK, E. and D. MCDERMOTT. *Introduction to Artificial Intelligence.* Addison-Wesley (Reading (Mass.)), 1986.

[8] CLAUS, N. (= E. LUCAS). *La Tour d'Hanoï, Véritable casse-tête annamite.* P. Bousrez (Tours), 1883.

[9] —— La Tour d'Hanoï, Jeu de calcul. *Science et Nature, Vol. I, N° 8* (1884), 127-128.

[10] CROWE, D. W. The *n*-dimensional cube and the Tower of Hanoi. *Amer. Math. Monthly 63* (1956), 29-30.

[11] CULL, P. and E. F. ECKLUND Jr. On the Towers of Hanoi and Generalized Towers of Hanoi Problems. *Congress. Numer. 35* (1982), 229-238.

[12] —— Towers of Hanoi and analysis of algorithms. *Amer. Math. Monthly 92* (1985), 407-420.
[13] CULL, P. and C. GERETY. Is Towers of Hanoi Really Hard? *Congress. Numer. 47* (1985), 237-242.
[14] DOMORYAD, A. P. *Mathematical Games and Pastimes.* Pergamon Press (Oxford), 1964.
[15] DUDENEY, H. E. *The Canterbury Puzzles (and other curious problems).* E. P. Dutton (New York), 1908.
[16] ER, M. C. An iterative solution to the generalized Towers of Hanoi problem. *BIT 23* (1983), 295-302.
[17] —— An analysis of the generalized Towers of Hanoi Problem. *BIT 23* (1983), 429-435.
[18] —— The Generalized Towers of Hanoi Problem. *J. Inform. Optim. Sci. 5* (1984), 89-94.
[19] FERMAT, P. DE. *Œuvres, Tome Deuxième (Correspondance).* Gauthier-Villars (Paris), 1894.
[20] FRAME, J. S. (Solution to Problem 3918). *Amer. Math. Monthly 48* (1941), 216-217.
[21] GILLISPIE, C. G. (ed.). *Dictionary of scientific biography, Volume VIII.* Scribner's Sons (New York), 1973.
[22] GRAHAM, R. L., D. E. KNUTH and O. PATASHNIK. *Concrete Mathematics.* Addison-Wesley (Reading (Mass.)), 1989.
[23] GRAVES, R. P. *Life of Sir William Rowan Hamilton, Vol. III.* Dublin University Press (Dublin), 1889.
[24] HARKIN, D. On the mathematical work of François-Edouard-Anatole Lucas. *Enseign. Math. (2) 3* (1957), 276-288.
[25] HERING, H. Zur Mathematisierung des "Turm von Hanoi"-Spiels. *MNU 26* (1973), 408-411.
[26] HINZ, A. M. An Iterative Algorithm for the Tower of Hanoi with Four Pegs. *Computing 42* (1989), 133-140.
[27] JOHNSONBAUGH, R. *Essential Discrete Mathematics.* Macmillan/Collier Macmillan (New York/London), 1987.
[28] LAVALLÉE, I. Note sur le problème des Tours de Hanoï. *Rev. Roumaine Math. Pures Appl. 30* (1985), 433-438.
[29] LONGCHAMPS, G. DE. Variétés. *J. Math. Spéciales (2) 2* (1883), 286-287.
[30] LUCAS, E. *Récréations mathématiques.* Gauthier-Villars (Paris), 1882.
[31] —— *(Discours prononcé à la distribution solennelle des prix, faite le mardi 4 août 1885).* Lycée Saint-Louis (Paris), 1885.
[32] —— *Nouveaux jeux scientifiques de M. Edouard Lucas.* La Nature 17 (1889), 2e semestre, 301-303.
[33] —— *Récréations mathématiques, tome III.* Gauthier-Villars (Paris), 1893.
[34] LUNNON, W. F. The Reve's Puzzle. *Comput. J. 29* (1986), 478.
[35] MATTHES, G. Der Einsatz des "Turm von Hanoi"-Computerprogramms zur Diagnostik von Störungen des problemlösenden Denkens bei Patienten mit erworbenen Hirnschäden. *Biomedical J. 19* (1988), 10-13.
[36] PARVILLE, H. DE. La Tour d'Hanoï et la Question du Tonkin. *La Nature 12* (1884), 285-286.
[37] PAULL, M. C. *Algorithm design, A Recursion Transformation Framework.* Wiley (New York), 1988.
[38] PETTOROSSI, A. Towers of Hanoi problems: deriving iterative solutions by program transformations. *BIT 25* (1985), 327-334.
[39] ROHL, J. S. *Recursion via Pascal.* Cambridge University Press (Cambridge), 1984.

[40] ROHL, J. S. and T. D. GEDEON. The Reve's Puzzle. *Comput. J. 29* (1986), 187-188; Corrigendum *31* (1988), 190.
[41] ROTH, T. The Tower of Brahma revisited. *J. Recreational Math. 7* (1974), 116-119.
[42] SCARIONI, F. and H. G. SPERANZA. A probabilistic analysis of an error-correcting algorithm for the Towers of Hanoi puzzle. *Inform. Process. Lett. 18* (1984), 99-103.
[43] SCHOUTE, P. H. De Ringen van Brahma. *Eigen Haard* 1884, 274-276.
[44] SIMON, H. A. The Functional Equivalence of Problem Solving Skills. *Cognitive Psychology 7* (1975), 268-288.
[45] STEWART, B. M. (Solution to Problem 3918). *Amer. Math. Monthly 48* (1941), 217-219.
[46] TANNERY, P. *Mémoires scientifiques, Vol. VI.* Privat/Gauthier-Villars (Toulouse/ Paris), 1926.
[47] WAERDEN, B. L. van der. *Science Awakening.* Noordhoff (Groningen), 1954.
[48] WALSH, T. R. A case for iteration. *Congress. Numer. 40* (1983), 409-417.
[49] WOOD, D. The Towers of Brahma and Hanoi revisited. *J. Recreational Math. 14* (1981-82), 17-24.
[50] —— Adjudicating a Towers of Hanoi Contest. *Internat. J. Comput. Math. 14* (1983), 199-207.

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