

Zeitschrift:	L'Enseignement Mathématique
Herausgeber:	Commission Internationale de l'Enseignement Mathématique
Band:	35 (1989)
Heft:	1-2: L'ENSEIGNEMENT MATHÉMATIQUE
Artikel:	AN ALGEBRAIC PROOF OF VAN DER WAERDEN'S THEOREM
Autor:	Bergelson, Vitaly / Furstenberg, Hillel / Hindman, Neil / Katznelsion, Yitzhak
Bibliographie	
DOI:	https://doi.org/10.5169/seals-57373

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Siehe Rechtliche Hinweise.

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. Voir Informations légales.

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. See Legal notice.

Download PDF: 22.05.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

Proof. By Lemma 2.4 $M(\beta\mathbf{N}) \neq \emptyset$ so pick $p \in M(\beta\mathbf{N})$ and let $\mathbf{p} = (p, p, \dots, p)$. By Lemma 1.2 pick $i \in \{1, 2, \dots, m\}$ such that $cl A_i$ is a neighborhood of p and let $U = cl A_i$. Then $U \times U \times \dots \times U$ is a neighborhood of \mathbf{p} while, by Theorem 2.3, $\mathbf{p} \in I = cl I^*$. Pick $a, d \in \mathbf{N}$ with $(a, a+d, a+2d, \dots, a+(l-1)d) \in U \times U \times \dots \times U$. Then

$$\{a, a+d, a+2d, \dots, a+(l-1)d\} \subseteq U \cap \mathbf{N} = (cl A_i) \cap N = A_i. \quad \square$$

We remark that if one starts with the free semigroup on l letters in place of \mathbf{N} , essentially the same proof yields the Hales-Jewett Theorem. See [3] for the details.

REFERENCES

- [1] BERGLUND, J., H. JUNGHENN and P. MILNES. *Compact right topological semigroups and generalizations of almost periodicity*. Lecture Notes in Math. 663, Springer-Verlag, Berlin (1978).
- [2] ELLIS, R. *Lectures on Topological Dynamics*. Benjamin, New York, 1969.
- [3] FURSTENBERG, H. and Y. KATZNELSON. Idempotents and coloring theorems. To appear.
- [4] FURSTENBERG, H. and B. WEISS. Topological dynamics and combinatorial number theory. *J. d'Analyse Math.* 34 (1978), 61-85.
- [5] GRAHAM, R. and B. ROTHSCHILD. A short proof of van der Waerden's Theorem. *Proc. Amer. Math. Soc.* 42 (1974), 385-386.
- [6] HINDMAN, N. Ultrafilters and Ramsey Theory — an update. *Proceedings of "Set Theory and its Applications – Conference at York"*. To appear.
- [7] SHELAH, S. Primitive recursive bounds for van der Waerden numbers. *J. Amer. Math. Soc.* 1 (1989), 683-697.
- [8] TAYLOR, A. A note on van der Waerden's Theorem. *J. Comb. Theory (Series A)* 33 (1982), 215-219.
- [9] VAN DER WAERDEN, B. Beweis einer Baudetschen Vermutung. *Nieuw Arch. Wisk* (3) 19 (1927), 212-216.

(Reçu le 14 juin 1989)

Vitaly Bergelson

Department of Mathematics
Ohio State University
Columbus, OH 43210
(USA)

Hillel Furstenberg

Department of Mathematics
Hebrew University
Jerusalem (Israel)

Neil Hindman

Department of Mathematics
Howard University
Washington, DC 20059
(USA)

Yitzhak Katznelson

Department of Mathematics
Stanford University
Stanford, CA 94305
(USA)

vide-leer-empty