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Artikel: ABOUT THE PROOFS OF CALABI'S CONJECTURES ON COMPACT KÄHLER MANIFOLDS

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We obtain,

$$-\Delta'(S_{n,n}+c_7 S_{n-1,n-1}) \geqslant (c_1 c_7 - c_4^2 - c_5)S_{n,n} - (c_6 + c_2 c_7)(S_{n,n})^{\frac{1}{2}} - c_3 c_7$$

and the proof may be easily completed.

10. The analytic point of view

Since equation (1) is *elliptic* and g, as a Kähler metric, is real analytic for the underlying real (analytic) structure of X, by the general elliptic regularity theory e.g. [17], p. 266-277 if $P_\lambda(\varphi)$ is real analytic so are φ and g'. Hence a purely analytic proof would be desirable.

Real analytic inverse function theorems are available since the work of J. Nash [19] who made a decisive use of smoothing operators (see also [13]). A theorem of H. Jaccbowitz [15] (p. 203) (see also [25], p. 94-101, 137-138) is available, the proof of which is purely analytical and does not use smoothing operators. This approach was first initiated by A. Kolmogorov (1954) and developed by V. Arnold (1961) (see references in [18]), and by J. Moser [18] (p. 513-533). Unfortunately, the application to nonlinear elliptic operators is not achieved.

A further trouble arises from the fact that the space of analytic functions *is not metrizable.*

Last but not least, we could not carry out analytic *a priori* estimates.

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