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Autor: Delanoë, Ph. / Hirschowitz, A.

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5. A PRIORI ESTIMATES: THE ORIGINAL WAY

According to proposition 4.3 we must prove now that, given any sequence of positive real numbers (K_i), $i \in \mathbf{N}$, there exists a sequence (C_i) such that

$$\forall i \in \mathbf{N}, \quad \| D^i P_\lambda(\varphi) \| \leqslant K_i$$

implies

$$\| \varphi \| \leqslant C_0, \quad \forall i \in \mathbf{N}, \quad \| D^i \nabla \bar{\nabla} \varphi \| \leqslant C_{i+2}.$$

These are *a priori* estimates of order zero, two, three, and so on .... In case $\lambda > 0$, the C^0 estimate is straightforward [2]. In case $\lambda = 0$, it becomes very tricky; proofs simpler than Yau's original one [24] (p. 352-359), based on the idea of uniformly estimating the $L^p(dX_g)$ norms of φ, may be found in [16] (dimension 2), [3] [21] and [4] (p. 148-149).

Estimates of order two and three are carried out by means of tensor calculus and of the Maximum Principle (for elliptic equations) [20] applied to *suitable* test functions. Though it is not everywhere clear in [21] [24], it is worth noting that the computations can be written intrinsically, i.e. without any reference to a *particular* system of coordinates (e.g. [2]), or even *coordinate free* (see section 6 below).

Further regularity is then recovered by Schauder theory e.g. [5] (lemma 1). In the sequel, we show how further estimates can be carried out instead, *just going ahead with coordinate free tensor calculus.* This occurs actually for any fully nonlinear second order elliptic equation on a compact Riemannian manifold, via a straightforward imitation of the device below.

Remark 5.1. It follows from the C^2 *a priori* estimates that the metrics g' are *a priori* uniformly equivalent to the original metric g (see e.g. [3], p. 75).

6. COORDINATE FREE TENSOR CALCULUS

Even coordinate free tensor calculus needs indices. Usually these indices refer to a *local* frame. Another way is to view these indices *globally* as labelling copies of the holomorphic and antiholomorphic tangent and cotangent bundles. From this point of view, a tensor written with indices is a section of the tensor product of a family of bundles indexed by an *unordered* set of indices (disregarding those indices subject to the summation convention).