

<b>Zeitschrift:</b>	L'Enseignement Mathématique
<b>Herausgeber:</b>	Commission Internationale de l'Enseignement Mathématique
<b>Band:</b>	34 (1988)
<b>Heft:</b>	1-2: L'ENSEIGNEMENT MATHÉMATIQUE
 <b>Artikel:</b>	ABOUT THE PROOFS OF CALABI'S CONJECTURES ON COMPACT KÄHLER MANIFOLDS
<b>Autor:</b>	Delanoë, Ph. / Hirschowitz, A.
<b>Kapitel:</b>	3. Local inversion
<b>DOI:</b>	<a href="https://doi.org/10.5169/seals-56591">https://doi.org/10.5169/seals-56591</a>

### Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Siehe Rechtliche Hinweise.

### Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. Voir Informations légales.

### Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. See Legal notice.

**Download PDF:** 24.05.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

LEMMA 2.1. Let  $A, B$  be metric spaces, with  $A \neq \emptyset$  and  $B$  connected. Let  $P: A \rightarrow B$  be a continuous map. Assume:

- (i)  $P$  is open,
- (ii)  $P$  is proper, that is, for any compact subset  $K$  in  $B$ ,  $P^{-1}(K)$  is compact. Then  $P$  is surjective.

*Proof.* We only need to prove that  $P(A)$  is closed. Let  $b_0$  be a point in  $\overline{P(A)}$ . Since  $B$  is a metric space, there exists a sequence  $(b_i)_{i>0}$  in  $P(A)$  converging to  $b_0$ . The subset  $K = \{b_0, b_1, b_2, \dots\}$  is compact, hence so is  $PP^{-1}(K)$ . The latter contains  $b_1, \dots, b_i, \dots$ , hence  $b_0$ , and it is obviously contained in  $P(A)$ . Q.E.D.

In order to make use of this lemma, we shall need some inverse function theorem for (i), and some *a priori* estimates for (ii).

### 3. LOCAL INVERSION

THEOREM 3.1. Let  $X$  be a smooth compact manifold,  $V$  and  $W$  smooth vector bundles on  $X$ ,  $U$  an open set in  $C^\infty(X, V)$ , and  $P: U \rightarrow C^\infty(X, W)$ , a smooth nonlinear elliptic partial differential operator. Let  $A$  and  $B$  be LCFC submanifolds of  $U$  and of  $C^\infty(X, W)$  respectively, such that the restriction  $P_A$  of  $P$  to  $A$ , sends  $A$  into  $B$ . Then the Jacobian criterion holds for  $P_A$ , namely, if the derivative of  $P_A: A \rightarrow B$  is invertible at  $\varphi_0 \in A$ , then  $P_A$  is a local diffeomorphism near  $\varphi_0$ .

This is a convenient variant of the Nash-Moser theorem (e.g. [14]) regarding suitable restrictions of elliptic operators. It is established in a separate paper [11] (see also [22]). It relies only on the *classical* (Banach) inverse function theorem combined with *elliptic regularity*.

*Remark 3.2.* The Nash-Moser theorem has been studied by many authors, see the bibliography below and further references in [14] [15] [25].

### 4. PROPERNESS

In view of (2), theorem 3.1 implies that  $P_\lambda$  is open. We want to apply lemma 2.1 in order to prove that  $P_\lambda$  is surjective from  $A_\lambda$  to  $B_\lambda$ . Since  $P_\lambda(A_\lambda) \neq \emptyset$  (it contains 0), and since  $B_\lambda$  is connected, this amounts to proving that  $P_\lambda$  is *proper*. Let us explain why *a priori* estimates imply properness.

Concerning subsets in  $A_\lambda$  we have