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Artikel: ABOUT THE PROOFS OF CALABI'S CONJECTURES ON COMPACT KÄHLER MANIFOLDS

Autor: Delanoë, Ph. / Hirschowitz, A.

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LEMMA 2.1. *Let A, B be metric spaces, with $A \neq \emptyset$ and B connected. Let $P: A \to B$ be a continuous map. Assume:*

(i) *P is open,*

(ii) *P is proper, that is, for any compact subset K in $B, P^{-1}(K)$ is compact. Then P is surjective.*

Proof. We only need to prove that $P(A)$ is closed. Let b_0 be a point in $\overline{P(A)}$. Since B is a metric space, there exists a sequence $(b_i)_{i>0}$ in $P(A)$ converging to b_0. The subset $K = \{b_0, b_1, b_2, ...\}$ is compact, hence so is $PP^{-1}(K)$. The latter contains $b_1, ..., b_i, ...,$ hence b_0, and it is obviously contained in $P(A)$. Q.E.D.

In order to make use of this lemma, we shall need some inverse function theorem for (i), and some *a priori* estimates for (ii).

3. LOCAL INVERSION

THEOREM 3.1. *Let X be a smooth compact manifold, V and W smooth vector bundles on X, U an open set in $C^\infty(X, V)$, and $P: U \to C^\infty(X, W)$, a smooth nonlinear elliptic partial differential operator. Let A and B be LCFC submanifolds of U and of $C^\infty(X, W)$ respectively, such that the restriction P_A of P to A, sends A into B. Then the Jacobian criterion holds for P_A, namely, if the derivative of $P_A: A \to B$ is invertible at $\varphi_0 \in A$, then P_A is a local diffeomorphism near φ_0.*

This is a convenient variant of the Nash-Moser theorem (e.g. [14]) regarding suitable restrictions of elliptic operators. It is established in a separate paper [11] (see also [22]). It relies only on the *classical* (Banach) inverse function theorem combined with *elliptic regularity.*

Remark 3.2. The Nash-Moser theorem has been studied by many authors, see the bibliography below and further references in [14] [15] [25].

4. PROPERNESS

In view of (2), theorem 3.1 implies that P_λ is open. We want to apply lemma 2.1 in order to prove that P_λ is surjective from A_λ to B_λ. Since $P_\lambda(A_\lambda) \neq \emptyset$ (it contains 0), and since B_λ is connected, this amounts to proving that P_λ is *proper.* Let us explain why *a priori* estimates imply properness.

Concerning subsets in A_λ we have