Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 34 (1988)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: EULER'S FAMOUS PRIME GENERATING POLYNOMIAL AND THE CLASS NUMBER OF IMAGINARY QUADRATIC FIELDS

Autor: Ribenboim, Paulo

Bibliographie

DOI: https://doi.org/10.5169/seals-56587

$= a^2 + b^2p$, thus $1 > a^2 + (b^2-1)p$ and necessarily $a^2 = 0, b^2 = 1$, hence $4l = p$, which is absurd.

Now assume that there exists $m, 0 \leqslant m \leqslant q-2$, such that $f_q(m) = m^2 + m + q$ is not a prime. Then there exists a prime l such that $l^2 \leqslant m^2 + m + q$ and $m^2 + m + q = al$, with $a \geqslant 1$. Since $m^2 + m + q$ is odd then $l \neq 2$. Also $4l^2 \leqslant (2m+1)^2 + p < \left(\frac{p-1}{2}\right)^2 + p = \left(\frac{p+1}{2}\right)^2$, hence $l < \frac{p+1}{4} = q$. As it was shown, $\left(\frac{l}{p}\right) = -1$. However,

$$4al = (2m+1)^2 + 4q - 1 = (2m+1)^2 + p\,,$$

hence $-p$ is a square modulo l, so by Gauss' reciprocity law,

$$1 = \left(\frac{-p}{l}\right) = \left(\frac{-1}{l}\right)\left(\frac{p}{l}\right) = (-1)^{\frac{l-1}{2}}\left(\frac{l}{p}\right)(-1)^{\frac{l-1}{2} \times \frac{p-1}{2}} = \left(\frac{l}{p}\right),$$

and this is absurd. $\square$

BIBLIOGRAPHY

[1] Ayoub, R. and S. Chowla. On Euler's polynomial. *J. Nb. Th. 13* (1981), 443-445.

[2] Borevich, Z. I. and I. R. Shafarevich. *Number Theory*. Academic Press, New York, 1966.

[3] Cohn, H. *Advanced Number Theory*. Dover Publ., New York, 1962.

[4] Goldfeld, D. Gauss' class number problem for imaginary quadratic fields. *Bull. Amer. Math. Soc. 13* (1985), 23-37.

[5] Lehmer, D. H. On the function $x^2 + x + A$. *Sphinx 6* (1936), 212-214.

[6] Pritchard, P. A. Long arithmetic progressions of primes: some old, some new. *Math. of Comp. 45* (1985), 263-267.

[7] Rabinovitch, G. Eindeutigkeit der Zerlegung in Primzahlfaktoren in quadratischen Zahlkörper. *Intern. Congress of Math.*, Cambridge, 1912, vol. 1, 418-421.

[8] Ribenboim, P. *Algebraic Numbers*. Wiley-Interscience, New York, 1972.

[9] —— *The Book of Prime Number Records*. Springer Verlag, New York, 1988.

[10] Schinzel, A. and W. Sierpiński. Sur certaines hypothèses concernant les nombres premiers. Remarques. *Acta Arithm. 4* (1958), 185-208 and *5* (1959), p. 259.

[11] Schinzel, A. Remarks on the paper « Sur certaines hypothèses concernant les nombres premiers ». *Acta Arithm. 7* (1961), 1-8.

[12] Szekeres, G. On the number of divisors of $x^2 + x + A$. *J. Nb. Th. 6* (1984), 434-442.

(Reçu le 4 avril 1987)

Paulo Ribenboim

Queen's University
Kingston, Ontario
Canada K7L 3N6