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Artikel: EULER'S FAMOUS PRIME GENERATING POLYNOMIAL AND THE CLASS NUMBER OF IMAGINARY QUADRATIC FIELDS

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Bibliographie

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$= a^2 + b^2 p$, thus $1 > a^2 + (b^2-1)p$ and necessarily $a^2 = 0, b^2 = 1$, hence $4l = p$, which is absurd.

Now assume that there exists $m, 0 \leqslant m \leqslant q - 2$, such that $f_q(m) = m^2 + m + q$ is not a prime. Then there exists a prime l such that $l^2 \leqslant m^2 + m + q$ and $m^2 + m + q = al$, with $a \geqslant 1$. Since $m^2 + m + q$ is odd then $l \neq 2$. Also $4l^2 \leqslant (2m+1)^2 + p < \left(\frac{p-1}{2}\right)^2 + p = \left(\frac{p+1}{2}\right)^2$, hence $l < \frac{p+1}{4} = q$. As it was shown, $\left(\frac{l}{p}\right) = -1$. However,

$$4al = (2m+1)^2 + 4q - 1 = (2m+1)^2 + p\,,$$

hence $-p$ is a square modulo l, so by Gauss' reciprocity law,

$$1 = \left(\frac{-p}{l}\right) = \left(\frac{-1}{l}\right)\left(\frac{p}{l}\right) = (-1)^{\frac{l-1}{2}}\left(\frac{l}{p}\right)(-1)^{\frac{l-1}{2}\times\frac{p-1}{2}} = \left(\frac{l}{p}\right),$$

and this is absurd. □

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