

<b>Zeitschrift:</b>	L'Enseignement Mathématique
<b>Herausgeber:</b>	Commission Internationale de l'Enseignement Mathématique
<b>Band:</b>	34 (1988)
<b>Heft:</b>	1-2: L'ENSEIGNEMENT MATHÉMATIQUE
<b>Artikel:</b>	AN ELEMENTARY PROOF OF THE STRUCTURE THEOREM FOR CONNECTED SOLVABLE AFFINE ALGEBRAIC GROUPS
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<b>Bibliographie</b>	
<b>DOI:</b>	<a href="https://doi.org/10.5169/seals-56599">https://doi.org/10.5169/seals-56599</a>

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**THEOREM 6.** *If  $G = G_u \times T$  is a connected solvable affine group and  $S \subset G_s$  is a commuting set then  $Z_G(S)$  is connected and  $aSa^{-1} \subset T$  for some  $a \in G$ . In particular, all maximal tori of  $G$  are conjugate.*

*Proof.* We use induction on  $\dim G$ . The assertions are obvious if  $S \subset Z(G)$ . Otherwise choose  $s \in S \setminus Z(G)$ . By Theorem 5 we may assume that  $s \in T$ . Then  $Z_G(s)$  is a proper closed subgroup of  $G$  containing  $T$  and  $S$ . By Theorem 3,  $Z_G(s)$  is connected. Since  $\dim Z_G(s) < \dim G$ , we can apply the induction hypothesis to conclude the proof.  $\square$

It is now easy to describe connected nilpotent affine groups.

**THEOREM 7.** *A connected solvable affine group  $G = G_u \times T$  is nilpotent if and only if  $G_s = T \subset Z(G)$ . In that case  $G = G_u \times T$ .*

*Proof.* Assume that  $G$  is nilpotent. We prove that  $G_s = T \subset Z(G)$  by induction on  $\dim G$ . We may assume that  $G$  is not abelian. Let  $N$  be the last non-trivial term of the lower central series of  $G$ . Let  $f$  be as in Theorem 2 and  $\bar{G} = f(G)$ . Then  $\bar{G} = f(G_u T) = (\bar{G})_u f(T)$ . By induction hypothesis we have  $f(T) = (\bar{G})_s \subset Z(\bar{G})$ . Consequently if  $t \in T$  and  $x \in G$  then  $u := txt^{-1}x^{-1} \in N$ . Since  $N \subset Z(G) \cap G_u$ , and  $xtx^{-1} = u^{-1}t = tu^{-1}$  we must have  $u = 1$ . Thus  $T \subset Z(G)$  and, by Theorem 5,  $G_s = T$ . The converse is obvious.  $\square$

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(Reçu le 23 janvier 1988)

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