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satisfied. Let the the residue class field of $K(\varepsilon_n)$ have 2^k elements. Set $n' = (2^k)^{2^h} - 1$. Then $n \mid n'$, n' is odd, and $K(\varepsilon_{n'})/K(\varepsilon_n)$ is unramified of degree 2^h. Consider the conditions (i)-(v) with n' instead of n. Then (i) is unchanged, (ii) holds because $n \mid n'$, (iii) holds trivially and (v) holds vacuously because $2^h \mid (K(\varepsilon_{n'}): K)$. Finally $K(\varepsilon_{n'}) \cap K(\varepsilon_4) = K$ since one is ramified and the other is not, so the non-trivial automorphism of $K(\varepsilon_{4n})/K(\varepsilon_n)$ is the restriction of that of $K(\varepsilon_{4n'})/K(\varepsilon_{n'})$, so (iv) holds also for n'.

We can deduce from this abbreviated form of Janusz' theorem that it is equivalent to Yamada's. Suppose that Janusz' conditions are satisfied, and consider the extension $\mathbf{Q}_2(\varepsilon_{2^{h+1}}, \varepsilon_n)/K$. The inertia subgroup of its Galois group is $\mathscr{g} = \mathscr{G}(\mathbf{Q}_2(\varepsilon_{2^{h+1}}, \varepsilon_n)/K(\varepsilon_n))$, a group of order 4. Suppose that ρ is an extension of the non-trivial automorphism of $\mathbf{Q}_2(\varepsilon_{2^h}, \varepsilon_n)/K(\varepsilon_n)$ to $\mathbf{Q}_2(\varepsilon_{2^{h+1}}, \varepsilon_n)$, so $\rho \in \mathscr{g}$. By condition (iv), there is an integer $a \equiv -1 \pmod{2^h}$ such that $\rho(\varepsilon_{2^{h+1}}) = \varepsilon_{2^{h+1}}^a$. It follows that ρ^2 is the identity. Thus $\mathscr{g}$ is non-cyclic. Conversely suppose that there is an extension $\mathbf{Q}_2(\zeta)/K$ whose inertia subgroup $\mathscr{g}$ is non-cyclic. As we saw in 1., this means that σ_{-1} is in the Galois group of $\mathbf{Q}_2^c/K$ and so its restriction (which we also call σ_{-1}) is in $\mathscr{G}(\mathbf{Q}_2(\varepsilon_{2^h}, \varepsilon_c)/K)$ and is non-trivial. Its fixed field contains $K(\varepsilon_c)$; by Lemma 3.3 of [J], $K(\varepsilon_c, \varepsilon_4) = \mathbf{Q}_2(\varepsilon_{2^h}, \varepsilon_c)$ and so the fixed field is *exactly* $K(\varepsilon_c)$. Thus both (iv) and (ii) are also fulfilled. (i) holds by Lemma 1.

4. *F. Lorenz*, [*L*], *p. 463*. His condition for *non-triviality of* $S(K)$ is that -1 *is a norm in the extension* $K/\mathbf{Q}_2$. The norm residue symbol in the extension $\mathbf{Q}_2^c/\mathbf{Q}_2$ sends -1 to $\sigma_{-1} \in \mathscr{G}(\mathbf{Q}_2^c/\mathbf{Q}_2)$. Thus it follows from [S], pp. 204-205, that -1 is a norm in $K/\mathbf{Q}_2$ iff $\sigma_{-1} \in \mathscr{G}(\mathbf{Q}_2^c/K)$.

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