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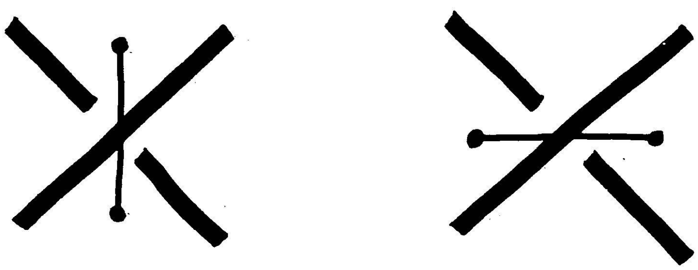
§ 3. Kauffman's state model for the Jones polynomial

Let K be a link diagram. By a state or a marker of K, we mean respectively a state or a marker of the corresponding link projection in R^2 (which is obtained from K by forgetting the overcrossing-undercrossing data). The markers of K are divided into two classes — positive and negative. By definition, if the over-line is rotated counterclockwise around the double point, then the first marker it meets is the positive one and the second one is negative:



positive marker negative marker

Figure 17

Let the diagram K be oriented. Consider the polynomial

$$V_K(t) = (-t)^{-3w(K)/4} \sum t^{(a_S - b_S)/4}(-t^{1/2} - t^{-1/2})^{|S|-1}$$

where $w(K)$ is the writhe number of K. The summation is over all the states S of K; the number of positive [respectively negative] markers of the state S is denoted by a_S [respectively b_S], and the number $|S|$ is defined in § 2.

It is shown in [5] that the polynomial $V_K(t)$ is equal to the Jones polynomial of the oriented link presented by K (see also [3]).

§ 4. Proof of Theorem 1

Orient the diagram K and denote the corresponding oriented link by L. Denote by A the state of K in which all markers are positive, and by $B = \check{A}$ the dual state in which all markers are negative. For any state S of K, denote by D_S and d_S respectively the maximal and minimal degrees in t in the expression

$$t^{(a_S - b_S)/4}(-t^{1/2} - t^{-1/2})^{|S|-1}$$