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LEMMA. *The vertices of the Newton polygon of f_n are*

$$(x_i, -\text{ord}_p(x_i!)), \quad 1 < i < s.$$

This follows easily from the previous lemmas.

It follows that the slopes of f are

$$m = \frac{-\text{ord}_p(x_i!) + \text{ord}_p(x_{i-1}!)}{x_i - x_{i-1}} = \frac{-(p^{n_i} - 1)}{p^{n_i}(p - 1)}.$$

COROLLARY A. *Suppose that p^m divides n . Then p^m divides the degree of each factor of f_n over \mathbf{Q}_p .*

Proof. Since p^m divides n , $m \leq n_s < n_{s-1} < \dots$. Hence, it follows from (1) that p^m divides the denominator of each m . Therefore the corollary follows from the corollary to Theorem NP.

COROLLARY B. *Suppose that $p^k \leq n$. Then p^k divides the degree of the splitting field of f_n over \mathbf{Q}_p .*

Proof. The hypotheses imply that $k \leq n_1$. Hence p^k divides the denominator of m_1 . As above this implies that p^k divides the degree of any extension of \mathbf{Q}_p formed by adjoining a root of f_n with valuation $-m_1$. This yields the corollary.

III. GLOBAL CONCLUSIONS A AND B

A. f_n is irreducible.

Suppose

$$n = \prod_p p^{n_p}$$

is the prime factorization of n . Corollary A implies that, for each prime p , p^{n_p} divides the degree of each factor of f_n over \mathbf{Q} . The conclusion follows.

B. Suppose $n/2 < p \leq n$ is a prime number. Then G contains a p -cycle.

By Corollary B, p divides the degree of the splitting field of f_n over \mathbf{Q}_p which divides the degree of the splitting field of f_n over \mathbf{Q} . Hence p divides the order of G_n . By Cauchy's Theorem G contains an element of order p . The conclusion follows since the only elements of order p in S_n are p -cycles if $p > n/2$.