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Artikel: ON THE GALOIS GROUPS OF THE EXPONENTIAL TAYLOR POLYNOMIALS

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LEMMA. *The vertices of the Newton polygon of* f_n *are*

$$(x_i, -\mathrm{ord}_p(x_i!)), \quad 1 < i < s.$$

This follows easily from the previous lemmas.

It follows that the slopes of f are

$$m = \frac{-\mathrm{ord}_p(x_i!) + \mathrm{ord}_p(x_{i-1}!)}{x_i - x_{i-1}} = \frac{-(p^{n_i}-1)}{p^{n_i}(p-1)}.$$

COROLLARY A. *Suppose that* p^m *divides* n. *Then* p^m *divides the degree of each factor of* f_n *over* $\mathbf{Q}_p$.

Proof. Since p^m divides n, $m \leqslant n_s < n_{s-1} < \dots$. Hence, it follows from (1) that p^m divides the denominator of each m. Therefore the corollary follows from the corollary to Theorem NP.

COROLLARY B. *Suppose that* $p^k \leqslant n$. *Then* p^k *divides the degree of the splitting field of* f_n *over* $\mathbf{Q}_p$.

Proof. The hypotheses imply that $k \leqslant n_1$. Hence p^k divides the denominator of m_1. As above this implies that p^k divides the degree of any extension of $\mathbf{Q}_p$ formed by adjoining a root of f_n with valuation $-m_1$. This yields the corollary.

III. GLOBAL CONCLUSIONS A AND B

A. f_n is irreducible.

Suppose

$$n = \prod_p p^{n_p}$$

is the prime factorization of n. Corollary A implies that, for each prime p, p^{n_p} divides the degree of each factor of f_n over $\mathbf{Q}$. The conclusion follows.

B. Suppose $n/2 < p \leqslant n$ is a prime number. Then G contains a p-cycle.

By Corollary B, p divides the degree of the splitting field of f_n over $\mathbf{Q}_p$ which divides the degree of the splitting field of f_n over $\mathbf{Q}$. Hence p divides the order of G_n. By Cauchy's Theorem G contains an element of order p. The conclusion follows since the only elements of order p in S_n are p-cycles if $p > n/2$.