

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 33 (1987)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** ON THE GALOIS GROUPS OF THE EXPONENTIAL TAYLOR POLYNOMIALS  
**Autor:** Coleman, Robert F.  
**Kapitel:** II. Application to the Exponential Taylor Polynomials  
**DOI:** <https://doi.org/10.5169/seals-87891>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 27.04.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

where the degree of  $g_i$  is  $x_i - x_{i-1}$  and all the roots of  $g_i(x)$  in  $\bar{\mathbf{Q}}_p$  have valuation  $-\left(\frac{y_i - y_{i-1}}{x_i - x_{i-1}}\right)$ .

We call the rational numbers,  $\frac{y_i - y_{i-1}}{x_i - x_{i-1}}$ , the slopes of  $g$ .

*Example.* The polynomial  $f_7$  has three factors over  $\mathbf{Q}_2$ , of degrees 4, 2 and 1, respectively, which have slopes  $-3/4$ ,  $-1/2$  and 0.

**COROLLARY.** Let  $d$  be a positive integer. Suppose that  $d$  divides the denominator of each slope (in lowest terms) of  $g$ . Then  $d$  divides the degree of each factor of  $g$  over  $\mathbf{Q}_p$ .

*Proof.* It suffices to show that  $d$  divides the degree of each irreducible factor of  $g$ . Let  $h$  be such a factor. Let  $\alpha \in \bar{\mathbf{Q}}_p$  be a root of  $h$ . Since  $d$  divides the denominator of the valuation of  $\alpha$  (by Theorem NP), it follows that  $d$  divides the index of ramification of the extension  $\mathbf{Q}_p(\alpha)/\mathbf{Q}_p$  which divides the degree of the extension which equals the degree of  $h$ .

## II. APPLICATION TO THE EXPONENTIAL TAYLOR POLYNOMIALS

Fix a prime number  $p$ .

**LEMMA.** Suppose  $k$  is a positive integer and

$$k = a_0 + a_1p + \dots + a_s p^s$$

where  $0 \leq a_i < p$ . Then

$$\text{ord}(k!) = \frac{k - (a_0 + a_1 + \dots + a_s)}{p - 1}.$$

This is easy and well known.

Now write

$$n = b_1 p^{n_1} + b_2 p^{n_2} + \dots + b_s p^{n_s}$$

where  $n_1 > n_2 > \dots > n_s$  and  $0 < b_i < p$ . Let

$$x_i = b_1 p^{n_1} + \dots + b_i p^{n_i}.$$

LEMMA. *The vertices of the Newton polygon of  $f_n$  are*

$$(x_i, -\text{ord}_p(x_i!)), \quad 1 < i < s.$$

This follows easily from the previous lemmas.

It follows that the slopes of  $f$  are

$$m = \frac{-\text{ord}_p(x_i!) + \text{ord}_p(x_{i-1}!)}{x_i - x_{i-1}} = \frac{-(p^{n_i} - 1)}{p^{n_i}(p - 1)}.$$

COROLLARY A. *Suppose that  $p^m$  divides  $n$ . Then  $p^m$  divides the degree of each factor of  $f_n$  over  $\mathbf{Q}_p$ .*

*Proof.* Since  $p^m$  divides  $n$ ,  $m \leq n_s < n_{s-1} < \dots$ . Hence, it follows from (1) that  $p^m$  divides the denominator of each  $m$ . Therefore the corollary follows from the corollary to Theorem NP.

COROLLARY B. *Suppose that  $p^k \leq n$ . Then  $p^k$  divides the degree of the splitting field of  $f_n$  over  $\mathbf{Q}_p$ .*

*Proof.* The hypotheses imply that  $k \leq n_1$ . Hence  $p^k$  divides the denominator of  $m_1$ . As above this implies that  $p^k$  divides the degree of any extension of  $\mathbf{Q}_p$  formed by adjoining a root of  $f_n$  with valuation  $-m_1$ . This yields the corollary.

### III. GLOBAL CONCLUSIONS A AND B

A.  $f_n$  is irreducible.

Suppose

$$n = \prod_p p^{n_p}$$

is the prime factorization of  $n$ . Corollary A implies that, for each prime  $p$ ,  $p^{n_p}$  divides the degree of each factor of  $f_n$  over  $\mathbf{Q}$ . The conclusion follows.

B. Suppose  $n/2 < p \leq n$  is a prime number. Then  $G$  contains a  $p$ -cycle.

By Corollary B,  $p$  divides the degree of the splitting field of  $f_n$  over  $\mathbf{Q}_p$  which divides the degree of the splitting field of  $f_n$  over  $\mathbf{Q}$ . Hence  $p$  divides the order of  $G_n$ . By Cauchy's Theorem  $G$  contains an element of order  $p$ . The conclusion follows since the only elements of order  $p$  in  $S_n$  are  $p$ -cycles if  $p > n/2$ .