Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 33 (1987)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: AN ORBIT CLOSING PROOF OF BROUWER'S LEMMA ON TRANSLATION ARCS

Autor: Fathi, Albert

Kurzfassung: Abstract

DOI: https://doi.org/10.5169/seals-87901

L'Enseignement Mathématique, t. 33 (1987), p. 315-322

AN ORBIT CLOSING PROOF OF BROUWER'S LEMMA ON TRANSLATION ARCS

by Albert FATHI [1])

ABSTRACT

We give a proof of the part of Brouwer's plane translation theorem which says that an orientation preserving homeomorphism of the plane with no fixed point cannot have a non wandering point.

There are several proofs of Brouwer's lemma on translation arcs-the cornerstone of his theorem on plane translation [Bu]. A recent one was given by Morton Brown [Bw1]. The proof given here is a better restatement of a proof I gave in a graduate course lecture in December 1980 in Orsay. The improvement is largely due to the interest shown by John Franks during our stay in Warwick in June 1986 for the special year on Smooth Ergodic Theory. In fact, John Franks' work on the Poincaré-Birkhoff theorem, see [F1] and [F2], contains a very nice use of Brouwer's lemma. One of the two main tricks used below is to use a global form of closing an orbit, the use of the local form of closing orbit is also one of the main ideas of [F1] and [F2]. I would like also to thank Lucien Guillou who introduced me to Brouwer's lemma and provided me with the first proof I ever understood of this lemma.

We recall a couple of definitions and notations. If $h: Z \to Z$ is a homeomorphism of the topological space Z, we denote by $\mathrm{Fix}(h)$ the set of fixed points of h and by $\mathrm{supp}(h)$ its support which is the closure in Z of the set $\{z \in Z \mid h(z) \neq z\}$.

[1]) Supported by NSF Grant # DMS-8610730(1).

Keywords: Brouwer, fixed point, homeomorphism, non wandering, plane, translation. *1980 Mathematics subject classifications:* 34C35, 54H20, 54H25, 55M20, 57N05.