Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 33 (1987)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: INVOLUTIONS IN SURFACE MAPPING CLASS GROUPS

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Kapitel: 3. Thurston's classification of mapping classes

DOI: https://doi.org/10.5169/seals-87897

By the same reasoning, we deduce that:

(15) $$t_b t_1^{-1} \in \Sigma,$$

(16) $$t_b t_2^{-1} \in \Sigma.$$

Hence, Humphries' generators are all conjugate modulo Σ. This implies that $M(F_g)/\Sigma$ is cyclic.

If the genus is greater than 2, equation (3) implies that $\Sigma = M(F_g)$. Since Σ is contained in $I(F_g)$, we see that $\Sigma = I(F_g) = M(F_g)$. Hence, theorem 1 is true for genus greater than two.

Now suppose that the genus is two. By equation (12), we conclude that i belongs to Σ. By lemma 2, it follows that $\Sigma = I(F_2)$. On the other hand, by equation (12), we conclude that

$$M(F_2)/\Sigma = Z_5.$$

This establishes theorem 1 for genus two.

This completes the proof of theorem 1.

3. Thurston's classification of mapping classes

The Teichmüller space of F, denoted by **T**, is the space of hyperbolic metrics on F up to isometry. It has a natural topology and is homeomorphic to an open ball of dimension $6g-6+2b$, where g is the genus of F and b the number of its boundary components.

Thurston's boundary of **T** is the space of projective classes of measured foliations on F.

A measured foliation is a foliation with isolated singularities of a special type (p-prong singularities, where p is any integer > 2, see figure 10), with a measure on transverse segments which is a Lebesgue-measure, and which is invariant by isotopy of the segment keeping each point on the same leaf.

There's an equivalence relation between measured foliations, generated by isotopy and the operation of collapsing a leaf connecting two singular points. **MF** denotes the space of equivalence classes.

There's a natural action on **MF** by the positive reals; **PMF** is the quotient projective space. **PMF** is homeomorphic to a sphere of dimension $6g-7+2b$ which constitutes, by Thurston's work, a natural boundary for Teichmüller space. $M(F)$, The mapping class group of F, acts continuously

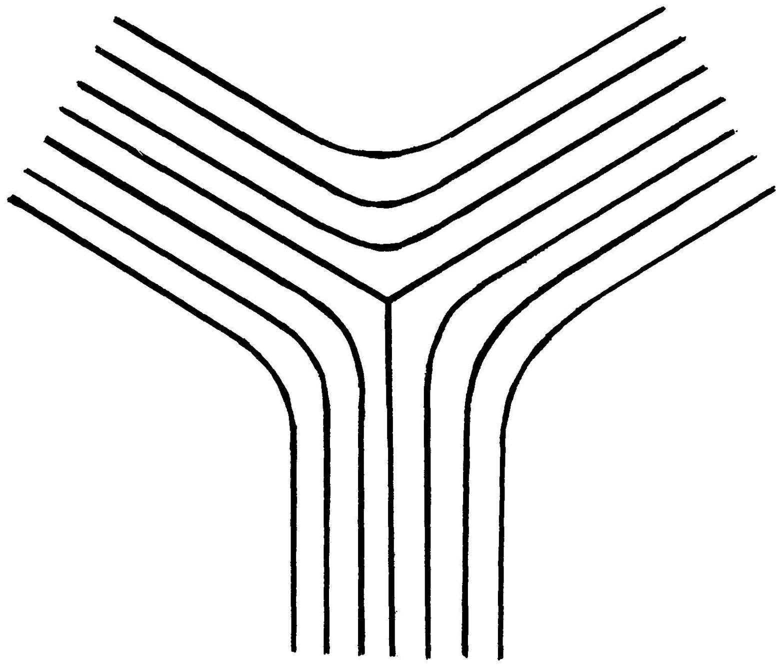

FIGURE 10.

on the closed ball $\mathbf{T} \cup \mathbf{PMF}$, and Thurston's classification of the elements of $M(F)$ can be formulated in terms of this action.

If an element of $M(F)$ has a fixed point in $\mathbf{T}$, then it is of finite order, i.e. there is an integer n such that the n-th iterate of that element is the class of the identity. In fact, there is a representative of this element which is globally periodic of order n, and which is an isometry of the hyperbolic metric corresponding to that fixed point in $\mathbf{T}$.

If an element of $M(F)$ does not have a fixed point in $\mathbf{T}$, then by the Brouwer fixed-point theorem it has a fixed point in $\mathbf{PMF}$.

There are two cases: either this point is the equivalence class of a foliation which has no closed cycles of leaves, and then this element is of *pseudo-Anosov* type, and can be represented by a homeomorphism of the surface which preserves a pair of measured foliations, acting as an expansion with respect to the transverse measure of one of them, and a contraction with respect to the other, or the fixed point in $\mathbf{PMF}$ is the class of a foliation which has a cycle of leaves; in this case the map is said to be *reducible*. There's an isotopy class of a (nonnecessarily connected) simple closed curve on M which is preserved by this mapping class, and the mapping class naturally splits into *components*.

We refer to [1] or [4] for the details of this classification.