# 2. What?

mathematics will play in this is as a mode of thought, a mental exercise, and an apprenticeship in rigour.

1.2.3. *Third approach:* the student has less need to *do* mathematics than to know how to read it. The professional literature is what will sustain his continuing development, much of it making use of mathematics. He must therefore be taught to study mathematics as a language rather than as a tool. He must be taught how to read it, to consult and use references. Mathematics assumes its important position as an element of culture and as a constantly developing science.

1.3. These three approaches lead, naturally, to different choices of content and teaching methods. We will return to this in later sections. Let us begin, however, with three opinions regarding why mathematics is taught to students of another discipline.

First opinion (expressed by students in economics at Budapest): the only justification for teaching mathematics is that it weeds out the bad students, because of the obstacle the mathematics examination presents.

Second opinion (expressed by mathematicians at Orsay): a justification for this teaching is that it teaches students how to use mathematics correctly and to distinguish, for example, how to construct a suitable model and to use the mathematical techniques associated with that model.

Third opinion (expressed by biologists at Orsay): it doesn't matter what mathematics is taught, if it is good mathematics; what is important is that students learn to reason mathematically.

Are these opinions completely idiosyncratic — or are they to be found expressed elsewhere?

## 2. What?

What mathematics should be taught?

2.1. A variety of very different possibilities arise depending upon the mathematical knowledge and understanding which students have gained at school. In some countries it may even be the case that students have opted out of school mathematics courses, and then find at university that their chosen subject, e.g. Biology, can have a considerable mathematical component. In certain cases, the initial goal of universities appears to be to bring all students to a common level through the teaching of basic techniques already met — but possibly not learned — at school. Where this goal is attained it raises questions concerning previous failures at the school level. Where failures occur the consequences are dramatic both for students and institutions (for example, in Florida, before they are allowed to enter the third year of a state university all students must pass a 'low-level' test in language and communication skills which depresses the standard of mathematics taught). At the other extreme, students enter university with a strong mathematical background, and are as well equipped to tackle new and demanding mathematics as those who have opted to become mathematicians (this is the case of many engineering students at Jadavpur University and of those entering the Ecole Supérieure d'Electricité at Orsay)[1]).

2.2. Current practice would appear to depend considerably upon national traditions. Thus at Southampton, second-year Physics students are taught partial differential equations, numerical analysis, tensors and finite group theory, none of which is taught at that stage to students at Orsay. However, third-year students at the latter institution meet Lebesgue integration, Hilbert spaces and Schwartz distributions, subjects not taught at Southampton (but in the syllabus at Eötvös Lorand University, Budapest).

How is one to explain such differences, and are they as irreconcilable as they at first sight appear?

2.3. We must draw attention here to two specific constraints on service teaching: the limited time available, and the fact that many students lack motivation. The former forces us to accept as axiomatic that service teaching can never supply students with *all* the mathematics they are likely to need.

2.4. Faced with these constraints the universities at Southampton and Orsay have adopted different attitudes.

2.4.1. *First attitude:* the primary purpose of mathematics service teaching is to acquaint the students with the mathematical techniques that will be useful or essential to them in their other courses and to give them some confidence in handling these techniques.

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[1]) That such students could follow any mathematics course reinforces the need to ask 'Why?' and 'What?' on their behalf. Although it lies outside the scope of this study, it is, of course, still essential continuously to pose the questions 'Why?', 'What?' and 'How?' in relation to all undergraduate courses in mathematics.

2.4.2. *Second attitude:* it is a matter of not elaborating and of moving quickly; for this one must emphasise modern and powerful tools and be prepared to forget about those tools whose life is limited — even if they are immediately usable in other course.

In practice, things are not so clearcut. The Southampton report gives as a secondary objective the need to give students an idea of the scope and power of mathematics, and to add to a 'utilitarian' approach certain 'cultural' overtones. At Orsay there is an insistence on the negotiation of programmes between mathematicians and other subject specialists — it is not sufficient to travel quickly, there must be agreement on the general direction.

2.5. The question of what one should teach gives rise to greater problems since it is inseparable from the questions 'who decides?' and 'who teaches?'.

2.5.1. The logic of the first attitude is that, as far as possible, it should be the teachers of the major discipline who teach the mathematical concepts which they will then use. They are aware of the needs, and the introduction of the mathematical ideas can be timed immediately to precede their application. This is the situation realised in Physics teaching at Cardiff and in Economics at the Karl Marx University, Budapest. The advantages are obvious: for coherence in teaching, motivation of students and a uniform use of language and symbolism. [1]) In fact the teachers' aims go beyond the utilitarian; for the physicists at Cardiff the mathematics must "help in the understanding of physical concepts and in the interpretation of experimental results" — criteria which have a fine ring, are all-embracing and are operable in all service teaching and do not exclude the cooperation of mathematicians. The engineers at Cardiff, however, see things somewhat differently. There the mathematics courses, jointly agreed and mainly classical, are given in the main by pure mathematicians, a state of affairs which the engineers do not find entirely satisfactory: "Engineering students should be taught by engineers, or at least by mathematicians who are based in the Engineering Faculty. The biggest single problem is motivation, and this is best achieved if the teaching is done by engineers who are respected by the students as engineers and who can draw examples to illustrate the mathematics from their own work... Mathematics for engineers *must* be taught as a means to an end and not as an intellectual discipline for its own sake and it is difficult for mathematicians to come to terms with this".

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[1]) An interesting consequence of this policy at Cardiff is that physicists are not specifically examined in mathematics: motivation for studying mathematics is intended to be gained from its teaching being so closely bound up with that of the physics.

2.5.2. The logic of the second attitude is to place responsibility in the hands of the mathematicians (the case, say, at Jadavpur). It is a question initially of identifying the needs of the major discipline. Following this the goal will be to model "non-mathematical situations in mathematical terms which apart from ensuring better insight into the situation involved, enables one to acquire a grip on problem-solving" and "to give a quantitative framework... a rational and scientific base". In every case, according to Jadavpur University, the mathematician must acquire the language of the [other] discipline, adapt it to a mathematical framework, provide a mathematical analysis, and then translate the results back into the user's language. Such a process, which is most ambitious and demands extremely strong interactions, is to be found at the research level between mathematicians and workers in other disciplines. Even though its realisation at a service teaching level might only be partial, it will have the advantage of permitting the mathematician to construct a coherent course with clearly identified goals. The duty of the mathematician is to construct the most straightforward and shortest course likely to attain these goals — in effect, what he is called upon to do in any course he gives. This might call for a wide knowledge of mathematics.

2.5.3. The two approaches are, in fact, compatible. Here, for example, we can quote a brave proposition advanced by E. Roubine (Ecole Supérieure d'Electricité) for the education of engineers. "Long term aims make it inevitable that there should be a break between mathematics and other teaching. It is reasonable to envisage a foundation course, relatively short, modern and at a high level, essentially of functional analysis (being built, today, upon numerical analysis). In other teaching one can devote a few lessons to reviewing other appropriate mathematics with the symbolism and language best suited to the immediate demands. Well carried out, this could suffice for the entire course." Thus algebra would naturally precede a course in computer science, statistics and probability those in agriculture, and coding theory one in telecommunications.

2.6. A strong argument for an initial mathematical education at a high level dissociated from immediate applications, is the power of computers. They demand that the user should become familiar with ever more sophisticated theories, for as Roubine demonstrates they now make available as everyday tools what were previously theories with little practical application. Thus, for example, Poincaré attempted to apply Fredholm theory of integral equations to aerials. Only, however, in the last ten years have engineers with the aid of computers been able to get to grips with singular integral equations.

2.7. Mathematical progress, and the revival of some older topics under the influence of the computer, force syllabus revisions. Pressures will also arise because of progress in the other disciplines (for example, the study of such complex phenomena as polymers and imperfect crystals). Here are a few specific questions.

2.7.1. What is the essential basic algebra and analysis which we should like all students to know? What can be acquired at a school level? What must wait until university?

2.7.2. What are the 'traditional' subjects which have been given new life by the computer and today's applications? A typical example arises from differential equations. "Special functions" are now scarcely taught to mathematicians, yet one finds them in the syllabus for chemistry students at Jadavpur. Does the role of symmetry in Physics and Chemistry suggest a place for 'classical groups and special functions'?

2.7.3. What geometry should be included? (The geologists at Budapest still hold on to traditional elementary geometry and descriptive geometry. Solid-state physicists and chemists are interested in polyhedra. Everywhere there are demands for geometric interpretations. Is there a case for introducing fractals and the corresponding mathematics (Weierstrass, Cantor, von Koch, Hausdorff...)?).

2.7.4. What is the place of statistics and probability? Should these be introduced piecemeal as needs arise, or presented as a structured course? The response may differ in, say, Physics, Biology and Economics. There have also been interesting experiments over some years in medical education.

2.7.5. What is the appropriate mathematics for computer scientists and who should teach it? Wouldn't its algebra, algorithmics and finite mathematics be equally appropriate for other students?

2.7.6. Several institutions now list 'operational research' as part of the mathematics syllabus. How should this be interpreted? Is OR, in fact, a part of mathematics or rather an independent (as yet minor) discipline which should itself be seen as being served by mathematics.

2.7.7. Extreme positions are expressed on certain topics for engineers, for example, Schwartz distributions: useless? Indispensable?

2.7.8. Is the teaching of mathematical modelling — 'a necessity' (Jadavpur) or 'a beautiful dream' (Budapest)?

## 3. How?

In the best possible way. And it could be argued that once it has been decided what should be taught and who should teach it, then it is a matter to be determined solely by the individuals concerned. There are, however, many general points which merit particular consideration.