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together with amended versions of the contributed papers will mark the end of the international stage of the study. We hope, however, that, as in the case of the computer study, particular aspects of the subject will then be examined in greater detail at regional and national meetings.

The Questions

Although it might not have universal acceptance, we shall take it as axiomatic that mathematics is taught as a service subject in response to a *need* (depending, naturally, on the major discipline concerned). What need? And what content and methods does this suggest? We propose to reflect on the three questions which arise (Why? What? How?) in the light of what might be done, of positive experiences encountered, and of open problems, rather than provide a simple description of the current stage of affairs.

1. Why?

Why do we teach mathematics to the students of discipline X?

There is no generally accepted answer to such a question. Of course, the responses will depend upon the particular discipline X, but we are also likely to obtain different responses from the specialists in X, from their students, and from the future employers of these students — each will hold different opinions.

1.1. In what way will mathematics be used in discipline X?

One example of a possible response is given by consideration of the award of the 1985 Nobel Prize for Chemistry to the two mathematicians, H. H. Hauptman and J. Karle, for their development of methods, based on Fourier analysis and probability, for determining crystal structures.[1])

In Physics, historical examples abound (Mechanics, Relativity, Quantum Theory). Currently, recourse to simulation on a computer has once again brought

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[1]) In W. Lipscomb's words, "The Nobel Prize for Chemistry is all about changing the field of chemistry. And this work changed the field".

together physicists and mathematicians, and has given new impetus to some mathematics so far little known (fractals). Informatics (computer science) could not be understood without mathematics, and the recent development of finite mathematics has been a direct response to its needs. Now Chemistry is beginning to rival Physics and Informatics as a valuable source of varied mathematical problems — as has just been shown by the award of the Nobel Prize to the crystallographers. In Medicine, specialists make use of sophisticated tools which necessitate interaction between them, physicists and engineers; mathematicians should play a part in the training which is needed. Biology and Economics are great users of statistical models. Linguistics, Geography and Geology have developed concepts and techniques which are made more readily accessible by a good mathematical understanding. Engineers, in all their branches of activity, have to calculate, to test hypotheses, and to construct models; must they be restricted in this to the use of traditional tools? On the other hand, is it possible for them to be acquainted with all the mathematics which could prove of use to them in their professional life? Recent events have shown that not all the mathematics which can be applied is to be found within that area conventionally called 'applied mathematics' (for example, algebra and theory of numbers have been utilised in coding theory and cryptography, algebraic topology in the chemistry of large molecules).

1.2. Since our teaching cannot encompass all the mathematics which might conceivably be used, what then are to be the criteria for selection?

1.2.1. *First approach:* the student must be capable of making use of those tools with which he is provided. He[1]) must therefore be restricted to concrete questions, techniques and concepts. The best motivation is supplied by considering examples drawn from his own discipline which can be solved using those mathematical techniques and concepts to which he has been introduced. He must shun abstract notions not immediately tied to applications.

1.2.2. *Second approach:* the student has at his disposal computers and software. This disposes of the need to teach many traditional techniques and skills, but creates a demand for other qualities. The student must know where to turn for help, what he can ask of the computer, and how to guide and control the machine. He must develop the knowledge and skills required to do this. The part

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[1]) For linguistic simplicity our typical student will be male. Nevertheless, we hope that this will not be interpreted as sexist. Certainly, in all countries there would seem to be a great need to increase the percentage of females studying those subjects which make heavy mathematical demands.

mathematics will play in this is as a mode of thought, a mental exercise, and an apprenticeship in rigour.

1.2.3. *Third approach:* the student has less need to *do* mathematics than to know how to read it. The professional literature is what will sustain his continuing development, much of it making use of mathematics. He must therefore be taught to study mathematics as a language rather than as a tool. He must be taught how to read it, to consult and use references. Mathematics assumes its important position as an element of culture and as a constantly developing science.

1.3. These three approaches lead, naturally, to different choices of content and teaching methods. We will return to this in later sections. Let us begin, however, with three opinions regarding why mathematics is taught to students of another discipline.

First opinion (expressed by students in economics at Budapest): the only justification for teaching mathematics is that it weeds out the bad students, because of the obstacle the mathematics examination presents.

Second opinion (expressed by mathematicians at Orsay): a justification for this teaching is that it teaches students how to use mathematics correctly and to distinguish, for example, how to construct a suitable model and to use the mathematical techniques associated with that model.

Third opinion (expressed by biologists at Orsay): it doesn't matter what mathematics is taught, if it is good mathematics; what is important is that students learn to reason mathematically.

Are these opinions completely idiosyncratic — or are they to be found expressed elsewhere?

2. What?

What mathematics should be taught?

2.1. A variety of very different possibilities arise depending upon the mathematical knowledge and understanding which students have gained at school. In some countries it may even be the case that students have opted out of school mathematics courses, and then find at university that their chosen subject, e.g. Biology, can have a considerable mathematical component. In certain cases, the initial goal of universities appears to be to bring all students to a common level through the teaching of basic techniques already met — but possibly not learned — at school. Where this goal is attained it raises questions concerning previous failures at the school level. Where failures occur the consequences are dramatic both for students and institutions (for example, in Florida, before they are allowed to enter the third year of a state university all students must pass a 'low-level' test in language and communication skills which depresses the standard of mathematics taught). At the other extreme, students enter university with a strong mathematical background, and are as well equipped to tackle new and demanding mathematics as those who have opted to become mathematicians (this is the case of many engineering students at Jadavpur University and of those entering the Ecole Supérieure d'Electricité at Orsay)[1]).

2.2. Current practice would appear to depend considerably upon national traditions. Thus at Southampton, second-year Physics students are taught partial differential equations, numerical analysis, tensors and finite group theory, none of which is taught at that stage to students at Orsay. However, third-year students at the latter institution meet Lebesgue integration, Hilbert spaces and Schwartz distributions, subjects not taught at Southampton (but in the syllabus at Eötvös Lorand University, Budapest).

How is one to explain such differences, and are they as irreconcilable as they at first sight appear?

2.3. We must draw attention here to two specific constraints on service teaching: the limited time available, and the fact that many students lack motivation. The former forces us to accept as axiomatic that service teaching can never supply students with *all* the mathematics they are likely to need.

2.4. Faced with these constraints the universities at Southampton and Orsay have adopted different attitudes.

2.4.1. *First attitude:* the primary purpose of mathematics service teaching is to acquaint the students with the mathematical techniques that will be useful or essential to them in their other courses and to give them some confidence in handling these techniques.

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[1]) That such students could follow any mathematics course reinforces the need to ask 'Why?' and 'What?' on their behalf. Although it lies outside the scope of this study, it is, of course, still essential continuously to pose the questions 'Why?', 'What?' and 'How?' in relation to all undergraduate courses in mathematics.

2.4.2. *Second attitude:* it is a matter of not elaborating and of moving quickly; for this one must emphasise modern and powerful tools and be prepared to forget about those tools whose life is limited — even if they are immediately usable in other course.

In practice, things are not so clearcut. The Southampton report gives as a secondary objective the need to give students an idea of the scope and power of mathematics, and to add to a 'utilitarian' approach certain 'cultural' overtones. At Orsay there is an insistence on the negotiation of programmes between mathematicians and other subject specialists — it is not sufficient to travel quickly, there must be agreement on the general direction.

2.5. The question of what one should teach gives rise to greater problems since it is inseparable from the questions 'who decides?' and 'who teaches?'.

2.5.1. The logic of the first attitude is that, as far as possible, it should be the teachers of the major discipline who teach the mathematical concepts which they will then use. They are aware of the needs, and the introduction of the mathematical ideas can be timed immediately to precede their application. This is the situation realised in Physics teaching at Cardiff and in Economics at the Karl Marx University, Budapest. The advantages are obvious: for coherence in teaching, motivation of students and a uniform use of language and symbolism.[1]) In fact the teachers' aims go beyond the utilitarian; for the physicists at Cardiff the mathematics must "help in the understanding of physical concepts and in the interpretation of experimental results" — criteria which have a fine ring, are all-embracing and are operable in all service teaching and do not exclude the cooperation of mathematicians. The engineers at Cardiff, however, see things somewhat differently. There the mathematics courses, jointly agreed and mainly classical, are given in the main by pure mathematicians, a state of affairs which the engineers do not find entirely satisfactory: "Engineering students should be taught by engineers, or at least by mathematicians who are based in the Engineering Faculty. The biggest single problem is motivation, and this is best achieved if the teaching is done by engineers who are respected by the students as engineers and who can draw examples to illustrate the mathematics from their own work... Mathematics for engineers *must* be taught as a means to an end and not as an intellectual discipline for its own sake and it is difficult for mathematicians to come to terms with this".

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[1]) An interesting consequence of this policy at Cardiff is that physicists are not specifically examined in mathematics: motivation for studying mathematics is intended to be gained from its teaching being so closely bound up with that of the physics.

2.5.2. The logic of the second attitude is to place responsibility in the hands of the mathematicians (the case, say, at Jadavpur). It is a question initially of identifying the needs of the major discipline. Following this the goal will be to model "non-mathematical situations in mathematical terms which apart from ensuring better insight into the situation involved, enables one to acquire a grip on problem-solving" and "to give a quantitative framework... a rational and scientific base". In every case, according to Jadavpur University, the mathematician must acquire the language of the [other] discipline, adapt it to a mathematical framework, provide a mathematical analysis, and then translate the results back into the user's language. Such a process, which is most ambitious and demands extremely strong interactions, is to be found at the research level between mathematicians and workers in other disciplines. Even though its realisation at a service teaching level might only be partial, it will have the advantage of permitting the mathematician to construct a coherent course with clearly identified goals. The duty of the mathematician is to construct the most straightforward and shortest course likely to attain these goals — in effect, what he is called upon to do in any course he gives. This might call for a wide knowledge of mathematics.

2.5.3. The two approaches are, in fact, compatible. Here, for example, we can quote a brave proposition advanced by E. Roubine (Ecole Supérieure d'Electricité) for the education of engineers. "Long term aims make it inevitable that there should be a break between mathematics and other teaching. It is reasonable to envisage a foundation course, relatively short, modern and at a high level, essentially of functional analysis (being built, today, upon numerical analysis). In other teaching one can devote a few lessons to reviewing other appropriate mathematics with the symbolism and language best suited to the immediate demands. Well carried out, this could suffice for the entire course." Thus algebra would naturally precede a course in computer science, statistics and probability those in agriculture, and coding theory one in telecommunications.

2.6. A strong argument for an initial mathematical education at a high level dissociated from immediate applications, is the power of computers. They demand that the user should become familiar with ever more sophisticated theories, for as Roubine demonstrates they now make available as everyday tools what were previously theories with little practical application. Thus, for example, Poincaré attempted to apply Fredholm theory of integral equations to aerials. Only, however, in the last ten years have engineers with the aid of computers been able to get to grips with singular integral equations.

2.7. Mathematical progress, and the revival of some older topics under the influence of the computer, force syllabus revisions. Pressures will also arise because of progress in the other disciplines (for example, the study of such complex phenomena as polymers and imperfect crystals). Here are a few specific questions.

2.7.1. What is the essential basic algebra and analysis which we should like all students to know? What can be acquired at a school level? What must wait until university?

2.7.2. What are the 'traditional' subjects which have been given new life by the computer and today's applications? A typical example arises from differential equations. "Special functions" are now scarcely taught to mathematicians, yet one finds them in the syllabus for chemistry students at Jadavpur. Does the role of symmetry in Physics and Chemistry suggest a place for 'classical groups and special functions'?

2.7.3. What geometry should be included? (The geologists at Budapest still hold on to traditional elementary geometry and descriptive geometry. Solid-state physicists and chemists are interested in polyhedra. Everywhere there are demands for geometric interpretations. Is there a case for introducing fractals and the corresponding mathematics (Weierstrass, Cantor, von Koch, Hausdorff...)?).

2.7.4. What is the place of statistics and probability? Should these be introduced piecemeal as needs arise, or presented as a structured course? The response may differ in, say, Physics, Biology and Economics. There have also been interesting experiments over some years in medical education.

2.7.5. What is the appropriate mathematics for computer scientists and who should teach it? Wouldn't its algebra, algorithmics and finite mathematics be equally appropriate for other students?

2.7.6. Several institutions now list 'operational research' as part of the mathematics syllabus. How should this be interpreted? Is OR, in fact, a part of mathematics or rather an independent (as yet minor) discipline which should itself be seen as being served by mathematics.

2.7.7. Extreme positions are expressed on certain topics for engineers, for example, Schwartz distributions: useless? Indispensable?

2.7.8. Is the teaching of mathematical modelling — 'a necessity' (Jadavpur) or 'a beautiful dream' (Budapest)?

3. How?

In the best possible way. And it could be argued that once it has been decided what should be taught and who should teach it, then it is a matter to be determined solely by the individuals concerned. There are, however, many general points which merit particular consideration.

3.1. *Statements and Proofs*

There can be no justification for giving statements which are incorrect, for example, for stating — or suggesting — that the Fourier series of a continuous function converges uniformly to that function. Yet there will be times when the teacher wishes to make statements because they are simple and correct in a convenient frame. For example, an integrable function on R tends to zero at infinity (in the sense of distributions). Each function on R is Lebesgue-measurable (in a model of set-theory which excludes the axiom of choice). Each part of a probability space is an event (in the same model). An essential point is to make useful statements in the most primitive possible language.

The choice of good definitions and statements is the work of a mathematician, but one in which non-mathematicians can usefully participate. It must also be recognised that there is nothing sacrosanct about the order in which material is presented. For example, it is not forbidden to define the rotation (curl) of a vector field starting out from a physical interpretation of Stokes' Theorem (Berkeley Physics Course), rather than from the usual operator definition in terms of derivatives: the theorem can precede the definition or vice-versa.

In a course given to mathematicians the guarantee of exactitude and of cohesion is the chain of logical argument, proof. In a service course then sometimes one must replace proof (too long, non-illuminating) by other arguments, and develop, for example, what George Polyá termed 'plausible reasoning'. Good physical illustrations can be more enlightening and impressive than proofs: depressing the sustaining key on a piano, saying 'ohh' to the strings, and hearing the response 'ohhhh' is an excellent gateway to spectral analysis and synthesis (Berkeley, *Waves,* p. 91).

On the other hand, exploratory work and verification on a computer can give certain mathematical statements the status of 'experimental' truths. Mathematical rigour consists in distinguishing between mathematical proof and experimental verification — this distinction must not become blurred.

3.2. *Examples and concepts*

Must one begin with examples and from these derive the concepts, or should one start off with the concepts and flesh these out with examples? This is an old question. Should one restrict oneself to examples drawn from the major discipline? Advice varies and depends upon many external constraints, in particular, the time available and class size.

One possibility merits special attention: this is the introduction of exploratory data analysis at the beginning of university studies. Manipulation can be done without any great theoretical apparatus; in addition, important motivation can be provided for the study of linear algebra and probability.

In general, the relations between examples, concepts and intuition generate major pedagogical questions. The great unifying concepts (groups, measure) are not accessible, despite their apparent simplicity, unless they are supported with numerous illustrations and examples. This is true of all mathematics teaching, but unfortunately within service teaching students are not provided with the time in which such notions can become familiar and intuitive.

3.3. *Small or large classes?*

Generally, the response to this question depends almost entirely on local resources: large groups demand fewer teachers. There are clear administrative advantages in the tradition of teaching service mathematics to large groups, often drawn from several different departments: economies of preparation of both lectures and exercises, and the possibility of employing only such lecturers as have a direct interest in service teaching and who, over the years, amass experience concerning likely points of difficulty, general needs etc. The disadvantages include the lack of motivation for the students, the restrictions placed on the kind of learning activities which can be offered, and the impossibility of setting a common examination which matches the real needs and strengths of students drawn from a range of departments. (We note, however, that at Eindhoven, even though the 'large group' format has been retained, this has not prevented the introduction of a novel course which depends upon each student having his own programmable pocket computer).

The question is also bound to that of 'who teaches?' The case for having a large inhomogeneous class taught by a mathematician is very strong. Small groups, on the other hand, are better able to utilise exercises and examples which draw on their major disciplines.

Even with any one discipline, however, first-year students are likely to differ very greatly in their mathematical attainments and abilities. This creates difficulties for the lecturer and forces consideration of other methods of teaching and learning. For such reasons, we should like our study to pay particular attention to experiments which have been made to help resolve such pedagogical problems. We note, for example, that at Southampton first-year engineers follow an individualised, 'self-paced' course based on reading (with frequent testing) rather than lectures.

3.4. *The 'Ideal' situation*

Subject to the various constraints which have to be met, what patterns of service teaching are giving rise to local satisfaction? We have already referred

(Section 2.5.1. above), to the way in which mathematics is taught to physicists at Cardiff. Here we give other examples of situations considered 'ideal'.

At Southampton, the course for chemists is given by a mathematician but each student, together with three or four others, is seen fortnightly by a chemist who will give tutorial supervision using material and example sheets supplied by the mathematician.[1]) A similar system has operated for some years in the Physics department at Orsay to general satisfaction: the lectures to the whole class being given by a mathematician, directed work (to groups of 20 students) by physicists.

At Paris-Grignon, a 20-hour course for third-year students of Agriculture was mounted in the form of a dialogue between an economist and a mathematician, thus providing the framework for an effective investigation. Such 'team-teaching' is very motivating for students, but is very expensive in preparation time.

No doubt other 'ideal' situations having different characteristics can be found. Detailed descriptions of them would be extremely welcome.

3.5. *The use of computers*

As was written above, the impact of computers on the teaching of mathematics has already been the subject of an ICMI study. It is essential that we reflect on all the new possibilities offered by computers (rapid computation, graphics, experimentation) and on the changing needs caused by their introduction (changes both of curricular content and also of desirable qualities to be developed in students.

A feature of the reports we received was the limited use of computers in the teaching of those subjects which have traditionally made heavy use of mathematics.

3.6. *The use of books and papers*

Here there are two aspects. First, for service teaching it is good to use texts written collaboratively by mathematicians and specialists in the major disciplines. Such books do exist, but there are many gaps. It would be valuable to have the characteristics of the successful texts, and also the lacunae, described.

Secondly, as we have already stressed, students must learn how to read mathematics, both in order to learn more mathematics when there is no lecturer

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[1]) One chemist wrote of this arrangement: 'many of my colleagues agree with me that in Southampton Chemistry we have the ideal situation as far as academic considerations are concerned. In tutorials the chemists can relate the material covered to Chemistry, point out the relevance to the Chemistry course and (it is hoped) provide some motivation'.

to hand, and also to understand their professional literature. Descriptions of 'planned' reading tasks are not numerous, but appear of interest and potential value (e.g. readings of extracts from Laplace for students at the Paris Ecole des Ponts et Chaussées, a chapter of Volterra for biologists at Orsay).

3.7. *Examinations, assessment and control*

In many cases examinations supply the principal motivation for students (although, as we have indicated in Section 2.5.1., this need not necessarily be the case). If the examination is outside the lecturer's control (as in Florida, and even more in the preparatory classes for the 'grandes écoles' in France), then it also provides motivation for him. Therefore, the questions 'Why?' and 'How?' should not be asked of teaching alone, but must also be asked of evaluation and assessment. If the teaching of mathematical modelling is a primary goal, then this goal is unlikely to be attained, if all that is required to pass the examination is memory of a ragbag of techniques applied in stock, purely mathematical situations. On the whole examinations tend to freeze courses, and militate against such innovations as, for example, the introduction of computers, mathematical modelling, and 'planned' reading. On the other hand, all of these innovations can be effectively examined, and examples can be given. However, their assessment is extremely time-consuming and the large numbers of students involved in service courses present particular difficulties.

How, then are we to use examinations and assessment as a means for *improving* teaching and learning? What desirable changes can be made to entrance examinations or to national examinations? Are there forms of continuous assessment which enable teachers/students to monitor the assimilation of the mathematics they teach/learn? Can this be done within the short time allocated to service teaching? Are there still examinations which contribute little and might be better abandoned? Examples of good practice will be welcomed.

4. CALL FOR PAPERS

In this discussion document is has been possible only briefly to indicate some questions of great interest and concern. The next step is to take a selection of these and to delve into them more deeply, to flesh arguments out with examples taken from current practice, to examine philosophical and pedagogical points more critically, to report the results of relevant research. The planning committee for the study would very much welcome papers which so develop points made in this paper, and which, in their turn, could form the bases of discussions in Udine in April, 1987. Such papers would be welcomed from all concerned with service teaching, mathematicians, specialists in other disciplines, students, recent students and employers.