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if $K(\alpha)$ has an odd [resp. even] number of components; in particular $V_\alpha(q)$ can be defined for any $q \in \mathbf{C}$, not just for those corresponding to good traces on some $\mathcal{A}_{\beta,n}$. And, most importantly for the early growth of the subject, a computation in the summer 1984 with the trefoil knot showed that V is not a mere variant of the Alexander polynomial. In fact, during a few hours, this was thought to reveal a mistake in computations! See end of § 7 for more details on the independence of the polynomials.

One way to recover the two variable polynomial is to introduce a family of traces on $H_{q,\infty} = \lim_{n \rightarrow \infty} H_{q,n}$, indexed by a complex parameter z . This programme was pursued by Ocneanu, and exposed in §§ 5-6 above. Observe that

- (1) Only one of Ocneanu's traces pass to the quotient $\mathcal{A}_{\beta,\infty}$, namely that corresponding to $z = q(q+1)^{-2}$.
- (2) Ocneanu's traces are positive for some values of the pair (q, z) only: the picture appears in Wenzl's thesis [We] and also in [Jo₄].
- (3) It does help to keep positivity considerations in mind when studying knot polynomials: see § 14 in [Jo₅].

ADDED IN PROOF

1. V. Turaev has another and simpler proof of some of the geometric arguments given in § 11. See a next issue of this journal.

2. K. Murasugi has informed us that he has now proved conjecture C.

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