

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 32 (1986)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON THE JONES POLYNOMIAL Swiss Seminar in Berne
Autor: de la Harpe, Pierre / Kervaire, Michel / Weber, Claude

Bibliographie

DOI: <https://doi.org/10.5169/seals-55091>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 18.08.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

if $K(\alpha)$ has an odd [resp. even] number of components; in particular $V_\alpha(q)$ can be defined for any $q \in \mathbb{C}$, not just for those corresponding to good traces on some $\mathcal{A}_{\beta,n}$. And, most importantly for the early growth of the subject, a computation in the summer 1984 with the trefoil knot showed that V is not a mere variant of the Alexander polynomial. In fact, during a few hours, this was thought to reveal a mistake in computations! See end of § 7 for more details on the independence of the polynomials.

One way to recover the two variable polynomial is to introduce a family of traces on $H_{q,\infty} = \lim_{n \rightarrow \infty} H_{q,n}$, indexed by a complex parameter z . This programme was pursued by Ocneanu, and exposed in §§ 5-6 above. Observe that

- (1) Only one of Ocneanu's traces pass to the quotient $\mathcal{A}_{\beta,\infty}$, namely that corresponding to $z = q(q+1)^{-2}$.
- (2) Ocneanu's traces are positive for some values of the pair (q, z) only: the picture appears in Wenzl's thesis [We] and also in [Jo₄].
- (3) It does help to keep positivity considerations in mind when studying knot polynomials: see § 14 in [Jo₅].

ADDED IN PROOF

1. V. Turaev has another and simpler proof of some of the geometric arguments given in § 11. See a next issue of this journal.
2. K. Murasugi has informed us that he has now proved conjecture C.

REFERENCES

- [Al] ALEXANDER, J. W. A lemma on a system of knotted Curves. *Proc. Nat. Acad. Sci. USA*. 9 (1923), 93-95.
- [Au] AUBERT, P. L. Projecteurs dans $\mathcal{U}(G)$: un exemple. *Lecture Notes in Math.* 725 Springer (1979), 17-18.
- [Ba] BANKWITZ, C. Über die Torsionszahlen der alternierenden Knoten. *Math. Ann.* 103 (1930), 145-161.
- [B.-Z.] BURDE, G. and H. ZIESCHANG. *Knots*. De Gruyter Studies in Mathematics (1985), 400 p.

- [Co₁] CONNES, A. Classification of injective factors. *Ann. of Math.* 104 (1976), 73-115.
- [Co₂] —— Classification des facteurs. *Proc. Symp. pure Math.* 38 (2) (AMS 1982), 43-109.
- [Cr] CROWELL, R. Non alternating links. *Illinois Jour. Math.* 3 (1959), 101-120.
- [De] DEHN, M. Die beide Kleeblattschlingen. *Math. Ann.* 75 (1914), 402-413.
- [Di] DIXMIER, J. *Les algèbres d'opérateurs dans l'espace hilbertien*, 2^e édition. Gauthier-Villars (1969).
- [FH] FACK, T. and P. DE LA HARPE. Sommes de commutateurs dans les algèbres de von Neumann finies continues. *Ann. Inst. Fourier* 30 (3) (1980), 49-73.
- [GHJ] GOODMAN, F., P. DE LA HARPE and V. JONES. Dynkin diagrams and towers of algebras. In préparation.
- [Jo₁] JONES, V. F. R. L'indice d'un sous-facteur d'un facteur de type II. *C.R. Acad. Sci. Série I*, 294 (1982), 391-394.
- [Jo₂] —— Index for subfactors. *Inventiones Math.* 72 (1983), 1-25.
- [Jo₃] —— A polynomial invariant for knots via von Neumann algebras. *Bulletin AMS* 12 (1985), 103-111.
- [Jo₄] —— A new knot polynomial and von Neumann algebras. *Notices AMS* 33 (1986), 219-225.
- [Jo₅] —— Hecke algebra representations of braid groups and link polynomials. To appear.
- [Ka₁] KAUFFMAN, L. The Conway polynomial. *Topology* 20 (1980), 101-108.
- [Ka₂] —— *Formal knot theory*. Math. Notes. Princeton University Press (1983), 168 p.
- [Ka₃] —— State models and the Jones polynomial. Preprint (1986), 30 p.
- [Kn] KNOTT, C. G. *Life and Scientific Work of P. G. Tait*. Cambridge University Press (1911).
- [Li] LITTLE, C. N. On knots, with a census for order ten. *Trans. Connecticut Acad. Sci.* 18 (1885), 374-378.
- [L.-M.] LICKORISH, W. B. R. and K. C. MILLETT. A polynomial invariant of oriented links. Preprint (1986), 80 p.
- [Ma] MAXWELL, J. C. *A treatise on electricity and magnetism*. Oxford (1883).
- [McD] McDUFF, D. Uncountably many II_1 -factors. *Ann. of Math.* 90 (1969), 372-377.
- [MN] MURRAY, F. J. and J. VON NEUMANN. On rings of operators IV. *Ann. of Math.* 44 (1944), 716-808.
- [Mo] MORTON, H. Threading knot diagrams. *Math. Proc. Camb. Phil. Soc.* 99 (1986), 247-260.
- [Mu₁] MURASUGI, K. On the genus of the alternating knot, I and II. *Jour. Math. Soc. Japan* 10 (1958), 94-105 and 235-248.
- [Mu₂] —— Jones polynomials and classical conjectures in knot theory. Preprint (1986), 21 p.
- [Pe] PERKO, K. On the classification of knots. *Proceedings AMS* 45 (1974), 262-266.
- [PP] PIMSNER, M. and S. POPA. Sur les sous-facteurs d'indice fini d'un facteur de type II_1 ayant la propriété T. *C. R. Acad. Sci. Série I*, 303 (1986), 359-361.
- [Sc] SCHWARTZ, J. Two finite, non hyperfinite, non isomorphic factors. *Comm. Pure Appl. Math.* 16 (1963), 19-26.
- [SZ] STRATILA, S. and L. ZSIDO. *Lectures on von Neumann algebras*. Abacus Press (1979). See also by S. STRATILA: *Modular theory in operator algebras*. Abacus Press.

- [Tai] TAIT, P. G. On Knots I, II, III. *Scientific papers Vol. 1* (1898), 273-347.
- [Tak] TAKESAKI, M. *Theory of operator algebras I*. Springer (1979).
- [Thi] THISTLETHWAITE, M. B. A spanning tree expansion of the Jones polynomial. Preprint (1986), 21 p.
- [Ti] TIETZE, H. Über die topologischen Invarianten mehrdimensionaler Mannigfaltigkeiten. *Monatshefte für Math. u. Phys.* 19 (1908), 1-118.
- [We] WENZL, H. *Representations of Hecke algebras and subfactors*. Thesis, University of Pennsylvania, 1985.

(Reçu le 21 juillet 1986)

Pierre de la Harpe

Michel Kervaire

Claude Weber

Section de mathématiques, Université de Genève

Case postale 240

CH-1211 Genève 24