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produces a path with the same tail as $\overrightarrow{v\,gv}$. The process then continues as for g and

$$\psi(\gamma_f g) = \lambda_f\, a_{x_1}\, \lambda_{f_1} \dots a_{x_r}\, \lambda_{f_r}\, a_v R = \psi(\gamma_f)\psi(g)\,.$$

Otherwise $\overrightarrow{v\,gv}$ contains $\overrightarrow{vy}$. Then $x_1 = z$, $\gamma_{f_1} \doteq \gamma_{\bar f}$ and we may as well take $a_{x_1} = 1$. A first tail wag of $\overrightarrow{v\,gv}$ using γ_f leaves a path with the same tail as $\overrightarrow{v(\gamma_f g)v}$. Thus

$$\begin{aligned}\psi(\gamma_f g) &= a_{x_2}\, \lambda_{f_2} \dots a_{x_r}\, \lambda_{f_r}\, a_v R \\ &= \lambda_f\, \lambda_{\bar f}\, a_{x_2}\, \lambda_{f_2} \dots a_{x_r}\, \lambda_{f_r}\, a_v R \\ &= \psi(\gamma_f)\psi(g)\,.\end{aligned}$$

This completes the proof that ψ is a homomorphism.

Our construction of ψ ensures that if $\psi(g) = R$ then $g = 1$. So ψ is injective. The cosets $h_w R$ (w a vertex of T and $h(w) = w$) and $\lambda_f R$ (f an edge of X/G) together generate $[(*\, G_w) * F]/R$. Now $\psi(h) = h_x R$ where x is the nearest fixed point of h to v. But h fixes all of $\overrightarrow{xw}$ so

$$\psi(h) = h_x R = h_w R\,.$$

Also

$$\psi(\gamma_f) = \lambda_f R\,.$$

Therefore the image of ψ is all of $[(*\, G_w) * F]/R$ and we have shown that ψ is an isomorphism.

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