Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 32 (1986)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: TREES, TAIL WAGGING AND GROUP PRESENTATIONS
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Bibliographie
DOI: https://doi.org/10.5169/seals-55090

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produces a path with the same tail as $\overrightarrow{v\,gv}$. The process then continues as for g and

$$\psi(\gamma_f g) = \lambda_f\, a_{x_1}\, \lambda_{f_1} \dots a_{x_r}\, \lambda_{f_r}\, a_v R = \psi(\gamma_f)\psi(g)\,.$$

Otherwise $\overrightarrow{v\,gv}$ contains $\overrightarrow{vy}$. Then $x_1 = z$, $\gamma_{f_1} = \gamma_{\bar{f}}$ and we may as well take $a_{x_1} = 1$. A first tail wag of $\overrightarrow{v\,gv}$ using γ_f leaves a path with the same tail as $\overrightarrow{v(\gamma_f g)v}$. Thus

$$\begin{aligned}\psi(\gamma_f g) &= a_{x_2}\, \lambda_{f_2} \dots a_{x_r}\, \lambda_{f_r}\, a_v R \\ &= \lambda_f\, \lambda_{\bar{f}}\, a_{x_2}\, \lambda_{f_2} \dots a_{x_r}\, \lambda_{f_r}\, a_v R \\ &= \psi(\gamma_f)\psi(g)\,.\end{aligned}$$

This completes the proof that ψ is a homomorphism.

Our construction of ψ ensures that if $\psi(g) = R$ then $g = 1$. So ψ is injective. The cosets $h_w R$ (w a vertex of T and $h(w) = w$) and $\lambda_f R$ (f an edge of X/G) together generate $[(*\,G_w)*F]/R$. Now $\psi(h) = h_x R$ where x is the nearest fixed point of h to v. But h fixes all of $\overrightarrow{xw}$ so

$$\psi(h) = h_x R = h_w R\,.$$

Also

$$\psi(\gamma_f) = \lambda_f R\,.$$

Therefore the image of ψ is all of $[(*\,G_w)*F]/R$ and we have shown that ψ is an isomorphism.

The author would like to thank the members of the Mathematics Department of the University of Geneva for their hospitality during the preparation of this article.

REFERENCES

[1] DICKS, W. *Groups Trees and Projective Modules.* Lect. Notes in Math. 790, Springer-Verlag 1980.

[2] HAUSMANN, J.-C. Sur l'usage de critères pour reconnaître un groupe libre, un produit amalgamé ou une *HNN*-extension. *L'Enseignement Mathématique* 27 (1981), 221-242.

[3] SCOTT, G. P. and C. T. C. WALL. Topological methods in group theory. *London Math. Soc. Lect. Notes 36*, Cambridge Univ. Press 1979, 137-203.
[4] SERRE, J.-P. *Arbres, Amalgames*, SL_2. Astérisque 46, Soc. Math. de France 1977.
[5] STALLINGS, J. R. Topology of finite graphs. *Inventiones Math. 71* (1983), 551-565.

(Reçu le 12 septembre 1985)

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