Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 32 (1986)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: TREES, TAIL WAGGING AND GROUP PRESENTATIONS

Autor: Armstrong, M. A.

Kapitel: 4. An inefficient choice

DOI: https://doi.org/10.5169/seals-55090

procedure. Since we shorten the tail at each step we eventually obtain a path which lies entirely in T and ends at say

$$(\gamma_{\bar{f}_r} a_{x_r}^{-1} \dots \gamma_{\bar{f}_2} a_{x_2}^{-1} \gamma_{\bar{f}_1} a_{x_1}^{-1} g)v \,.$$

Then $\gamma_{\bar{f}_r} a_{x_r}^{-1} \dots \gamma_{\bar{f}_1} a_{x_1}^{-1} g$ must fix v, say $\gamma_{\bar{f}_r} a_{x_r}^{-1} \dots \gamma_{\bar{f}_1} a_{x_1}^{-1} g = a_v \in G_v$. We now have

$$g = a_{x_1} \gamma_{f_1} \dots a_{x_r} \gamma_{f_r} a_v$$

and we somewhat optimistically define

$$\psi(g) = a_{x_1} \lambda_{f_1} \dots a_{x_r} \lambda_{f_r} a_v R \,.$$

4. An inefficient choice

Is ψ well defined? The geodesic from v to gv is certainly unique, as is the first point x_1 where it leaves T and its first edge e_m outside T. Both the edge e^1 and the group element γ_{f_1} are now determined by our original construction. The only ambiguity at this stage is the choice of the element $a_{x_1} \in G_{x_1}$ which maps e^1 to e_m. A different choice b_{x_1} will give a path from z_1 to $(\gamma_{\bar{f}_1} b_{x_1}^{-1} g)v$ which leaves T for the first time at say y_2. The first edge outside T will project to an edge f'_2 of X/G and so on until eventually we have g expressed as

$$g = b_{x_1} \gamma_{f_1} b_{y_2} \gamma_{f'_2} \dots b_{y_s} \gamma_{f'_s} b_v \,.$$

We must show that $a_{x_1} \lambda_{f_1} a_{x_2} \lambda_{f_2} \dots a_{x_r} \lambda_{f_r} a_v$ and $b_{x_1} \lambda_{f_1} b_{y_2} \lambda_{f'_2} \dots b_{y_s} \lambda_{f'_s} b_v$ determine the same left coset of R in $(\ast\, G_w) \ast F$.

Agree to select a_{x_1} from G_{x_1} so that the tail of the resulting path is as *long* as possible. Continue in this way selecting $a_{x_2}, a_{x_3} \dots$ so as to maximise the length of the tail at each stage. We shall compare any other set of choices with this rather inefficient selection.

Both a_{x_1} and b_{x_1} map e^1 to e_m, so $c = a_{x_1}^{-1} b_{x_1}$ must fix e^1. Also, due to our particular selection of a_{x_1}, the geodesic from z_1 to x_2 is left fixed by $\gamma_{\bar{f}_1} c \gamma_{f_1}$. Therefore

$$b_{x_1}\lambda_{f_1}b_{y_2}\lambda_{f'_2}\ldots b_{y_s}\lambda_{f'_s}b_v R$$
$$= a_{x_1}\lambda_{f_1}\lambda_{\bar{f}_1}a_{x_1}^{-1}b_{x_1}\lambda_{f_1}b_{y_2}\lambda_{f'_2}\ldots b_{y_s}\lambda_{f'_s}b_v R$$
$$= a_{x_1}\lambda_{f_1}\lambda_{\bar{f}_1}c_{x_1}\lambda_{f_1}b_{y_2}\lambda_{f'_2}\ldots b_{y_s}\lambda_{f'_s}b_v R$$
$$= a_{x_1}\lambda_{f_1}(\gamma_{\bar{f}_1}c\gamma_{f_1})_{z_1}b_{y_2}\lambda_{f'_2}\ldots b_{y_s}\lambda_{f'_s}b_v R$$
$$= a_{x_1}\lambda_{f_1}(\gamma_{\bar{f}_1}c\gamma_{f_1})_{x_2}b_{y_2}\lambda_{f'_2}\ldots b_{y_s}\lambda_{f'_s}b_v R$$
$$= a_{x_1}\lambda_{f_1}a'_{x_2}b_{y_2}\lambda_{f'_2}\ldots b_{y_s}\lambda_{f'_s}b_v R$$

where $a'_{x_2} = (\gamma_{\bar{f}_1}c\gamma_{f_1})_{x_2}$. If x_2 happens to equal y_2 then we simplify this further to

$$a_{x_1}\lambda_{f_1}a''_{x_2}\lambda_{f_2}b_{y_3}\lambda_{f'_3}\ldots b_{y_s}\lambda_{f'_s}b_v R$$

where a''_{x_2} is the product $a'_{x_2}b_{y_2}$ in G_{x_2}. We now compare a_{x_2} with a'_{x_2} if $x_2 \neq y_2$, noting that $\gamma_{f_2} = 1$ in this case, or with a''_{x_2} if $x_2 = y_2$, and repeat the process. Eventually we obtain

$$b_{x_1}\lambda_{f_1}b_{y_2}\lambda_{f'_2}\ldots b_{y_s}\lambda_{f'_s}b_v R = a_{x_1}\lambda_{f_1}a_{x_2}\lambda_{f_2}\ldots a_{x_r}\lambda_{f_r}a''_v R\,.$$

As $g = a_{x_1}\gamma_{f_1}\ldots a_{x_r}\gamma_{f_r}a_v = a_{x_1}\gamma_{f_1}\ldots a_{x_r}\gamma_{f_r}a''_v$ we see that $a''_v = a_v$. This completes the proof that ψ is well defined.

5. Nearest fixed points

To show ψ is a homomorphism we shall verify

$$\psi(hg) = \psi(h)\psi(g)$$

under the assumption that h either leaves some vertex of T fixed or is one of the elements γ_f. This is sufficient because the elements of the G_w (w a vertex of T) together with the γ_f (f an edge of $X/G-M$) form a set of generators for G.

Suppose h fixes the vertex w of T. Walk along the geodesic $\overrightarrow{vw}$ and let x be the first vertex we meet which is left fixed by h. Then $\overrightarrow{vx}$ is contained in T, and $\overrightarrow{vx}$ followed by $h(\overleftarrow{xv})$ is the geodesic from v to hv. This quits T for the first time at x and we see that

$$\psi(h) = h_x R\,.$$