

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 32 (1986)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: TREES, TAIL WAGGING AND GROUP PRESENTATIONS
Autor: Armstrong, M. A.
Kapitel: 4. An inefficient choice
DOI: <https://doi.org/10.5169/seals-55090>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 27.04.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

procedure. Since we shorten the tail at each step we eventually obtain a path which lies entirely in T and ends at say

$$(\gamma_{\bar{f}_r} a_{x_r}^{-1} \dots \gamma_{\bar{f}_2} a_{x_2}^{-1} \gamma_{\bar{f}_1} a_{x_1}^{-1} g)v.$$

Then $\gamma_{\bar{f}_r} a_{x_r}^{-1} \dots \gamma_{\bar{f}_1} a_{x_1}^{-1} g$ must fix v , say $\gamma_{\bar{f}_r} a_{x_r}^{-1} \dots \gamma_{\bar{f}_1} a_{x_1}^{-1} g = a_v \in G_v$.

We now have

$$g = a_{x_1} \gamma_{f_1} \dots a_{x_r} \gamma_{f_r} a_v$$

and we somewhat optimistically define

$$\psi(g) = a_{x_1} \lambda_{f_1} \dots a_{x_r} \lambda_{f_r} a_v R.$$

4. AN INEFFICIENT CHOICE

Is ψ well defined? The geodesic from v to gv is certainly unique, as is the first point x_1 where it leaves T and its first edge e_m outside T . Both the edge e^1 and the group element γ_{f_1} are now determined by our original construction. The only ambiguity at this stage is the choice of the element $a_{x_1} \in G_{x_1}$ which maps e^1 to e_m . A different choice b_{x_1} will give a path from z_1 to $(\gamma_{\bar{f}_1} b_{x_1}^{-1} g)v$ which leaves T for the first time at say y_2 . The first edge outside T will project to an edge f'_2 of X/G and so on until eventually we have g expressed as

$$g = b_{x_1} \gamma_{f_1} b_{y_2} \gamma_{f'_2} \dots b_{y_s} \gamma_{f'_s} b_v.$$

We must show that $a_{x_1} \lambda_{f_1} a_{x_2} \lambda_{f_2} \dots a_{x_r} \lambda_{f_r} a_v$ and $b_{x_1} \lambda_{f_1} b_{y_2} \lambda_{f'_2} \dots b_{y_s} \lambda_{f'_s} b_v$ determine the same left coset of R in $(*G_w)*F$.

Agree to select a_{x_1} from G_{x_1} so that the tail of the resulting path is as long as possible. Continue in this way selecting $a_{x_2}, a_{x_3} \dots$ so as to maximise the length of the tail at each stage. We shall compare any other set of choices with this rather inefficient selection.

Both a_{x_1} and b_{x_1} map e^1 to e_m , so $c = a_{x_1}^{-1} b_{x_1}$ must fix e^1 . Also, due to our particular selection of a_{x_1} , the geodesic from z_1 to x_2 is left fixed by $\gamma_{\bar{f}_1} c \gamma_{f_1}$. Therefore

$$\begin{aligned}
& b_{x_1} \lambda_{f_1} b_{y_2} \lambda_{f_2}' \dots b_{y_s} \lambda_{f_s}' b_v R \\
&= a_{x_1} \lambda_{f_1} \lambda_{\bar{f}_1} a_{x_1}^{-1} b_{x_1} \lambda_{f_1} b_{y_2} \lambda_{f_2}' \dots b_{y_s} \lambda_{f_s}' b_v R \\
&= a_{x_1} \lambda_{f_1} \lambda_{\bar{f}_1} c_{x_1} \lambda_{f_1} b_{y_2} \lambda_{f_2}' \dots b_{y_s} \lambda_{f_s}' b_v R \\
&= a_{x_1} \lambda_{f_1} (\gamma_{\bar{f}_1} c \gamma_{f_1})_{z_1} b_{y_2} \lambda_{f_2}' \dots b_{y_s} \lambda_{f_s}' b_v R \\
&= a_{x_1} \lambda_{f_1} (\gamma_{\bar{f}_1} c \gamma_{f_1})_{x_2} b_{y_2} \lambda_{f_2}' \dots b_{y_s} \lambda_{f_s}' b_v R \\
&= a_{x_1} \lambda_{f_1} a'_{x_2} b_{y_2} \lambda_{f_2}' \dots b_{y_s} \lambda_{f_s}' b_v R
\end{aligned}$$

where $a'_{x_2} = (\gamma_{\bar{f}_1} c \gamma_{f_1})_{x_2}$. If x_2 happens to equal y_2 then we simplify this further to

$$a_{x_1} \lambda_{f_1} a''_{x_2} \lambda_{f_2} b_{y_3} \lambda_{f_3}' \dots b_{y_s} \lambda_{f_s}' b_v R$$

where a''_{x_2} is the product $a'_{x_2} b_{y_2}$ in G_{x_2} . We now compare a_{x_2} with a'_{x_2} if $x_2 \neq y_2$, noting that $\gamma_{f_2} = 1$ in this case, or with a''_{x_2} if $x_2 = y_2$, and repeat the process. Eventually we obtain

$$b_{x_1} \lambda_{f_1} b_{y_2} \lambda_{f_2}' \dots b_{y_s} \lambda_{f_s}' b_v R = a_{x_1} \lambda_{f_1} a_{x_2} \lambda_{f_2} \dots a_{x_r} \lambda_{f_r} a''_v R.$$

As $g = a_{x_1} \gamma_{f_1} \dots a_{x_r} \gamma_{f_r} a_v = a_{x_1} \gamma_{f_1} \dots a_{x_r} \gamma_{f_r} a''_v$ we see that $a''_v = a_v$. This completes the proof that ψ is well defined.

5. NEAREST FIXED POINTS

To show ψ is a homomorphism we shall verify

$$\psi(hg) = \psi(h)\psi(g)$$

under the assumption that h either leaves some vertex of T fixed or is one of the elements γ_f . This is sufficient because the elements of the G_w (w a vertex of T) together with the γ_f (f an edge of $X/G-M$) form a set of generators for G .

Suppose h fixes the vertex w of T . Walk along the geodesic \overrightarrow{vw} and let x be the first vertex we meet which is left fixed by h . Then \overrightarrow{vx} is contained in T , and \overrightarrow{vx} followed by $h(\overrightarrow{xv})$ is the geodesic from v to hv . This quits T for the first time at x and we see that

$$\psi(h) = h_x R.$$