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§ 3. THE ALGEBRAIC PROPERTIES OF $\mathcal{E}$ (See [SKK], [Bj])

3.1. In the preceding section, we introduced the notion of micro-differential operators. The ring $\mathcal{E}$ of micro-differential operators has nice algebraic properties similar to those of the ring of holomorphic functions.

Let us recall some definitions of finiteness properties.

Definition 3.1.1. Let $\mathcal{A}$ be a sheaf of rings on a topological space S .

- (1) An $\mathcal{A}$ -module $\mathcal{M}$ is called *of finite type* (resp. *of finite presentation*) if for any point $x \in X$ there exists a neighborhood U and an exact sequence $0 \leftarrow \mathcal{M}|_U \leftarrow \mathcal{A}^p|_U$ (resp. $0 \leftarrow \mathcal{M}|_U \leftarrow \mathcal{A}^p|_U \leftarrow \mathcal{A}^q|_U$).
- (2) $\mathcal{M}$ is called *pseudo-coherent*, if any submodule of finite type defined on an open subset is of finite presentation. If $\mathcal{M}$ is pseudo-coherent and of finite type, then $\mathcal{M}$ is called *coherent*.
- (3) $\mathcal{M}$ is called *Noetherian* if $\mathcal{M}$ satisfies the following properties:
 - (a) $\mathcal{M}$ is coherent.
 - (b) For any $x \in X$, $\mathcal{M}_x$ is a Noetherian $\mathcal{A}_x$ -module (i.e. any increasing sequence of $\mathcal{A}_x$ -submodules is stationary).
 - (c) For any open subset U , any increasing sequence of coherent $(\mathcal{A}|_U)$ -submodules of $\mathcal{M}|_U$ is locally stationary.

As for the sheaf of holomorphic functions, we have

THEOREM 3.1.1 ([SKK] Chap. II, Thm. 3.4.1, Prop. 3.2.7). Let $\overset{\circ}{T}^*X$ denote the complement of the zero section in T^*X .

- (1) $\mathcal{E}_X$ and $\mathcal{E}_X(0)$ are Noetherian rings on T^*X .
- (2) $\mathcal{E}_X$ is flat over $\pi^{-1}\mathcal{D}_X$.
- (3) $\mathcal{E}_X(\lambda)|_{\overset{\circ}{T}^*X}$ is a Noetherian $\mathcal{E}_X(0)|_{\overset{\circ}{T}^*X}$ -module.
- (4) For $p \in T^*X$, $\mathcal{E}_X(0)_p$ is a local ring with the residual field $\mathbb{C}$.
- (5) A coherent $\mathcal{E}_X$ -module is pseudo-coherent over $\mathcal{E}_X(0)$.

§ 4. VARIANTS OF $\mathcal{E}$ (See [SKK], [Bj], [S])

4.1. We have defined the sheaf of rings $\mathcal{E}$. However we can introduce other sheaves of rings, similar to $\mathcal{E}$, which makes the theory transparent.