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Autor: Hildebrand, Adolf
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It would be interesting to determine those completely multiplicative functions $f(n) = \pm 1$, for which the analogue of the theorem does not hold. Schur [3] proved that if $f \not\equiv f_{\pm}$, where

$$f_{\pm}(n) = \begin{cases} (\pm 1)^k & \text{if} \quad n = 3^k m, m \equiv 1 \bmod 3\,, \\ -(\pm 1)^k & \text{if} \quad n = 3^k m, m \equiv 2 \bmod 3\,, \end{cases}$$

then there exists at least one $n \geqslant 1$, such that

$$f(n) = f(n+1) = f(n+2) = 1\,.$$

It is likely that under the same hypotheses there are infinitely many such n. Using arguments similar to those in section 3, one can prove this assertion under the additional hypotheses $f(2) = 1$ and $f(3) = -1$, but the general case seems to be more complicated.

A very plausible conjecture is that the integers n, for which (1) holds, have positive density. In the case $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$, this would follow from an analogous strengthening of the lemma by requiring (2) to hold on a set of positive density. Whereas a very simple argument shows that the equations $\lambda(n) = \lambda(n+1)$ and $\lambda(n+1) = \lambda(n-1)$ hold on a set of positive (lower) density (cf. [2]), this argument seems to break down, if n is required to lie in a prescribed residue class, and so far we have not been able to overcome this difficulty.

REFERENCES

[1] Chowla, S. *The Riemann hypothesis and Hilbert's tenth problem.* New York-London-Paris, Gordon and Breach 1965.
[2] Graham, S. and D. Hensley. Problem E 3025. *Am. Math. Monthly 90* (1983), 707.
[3] Schur, I. Multiplikativ signierte Folgen positiver ganzer Zahlen. In: *Gesammelte Abhandlungen von Issai Schur, Vol. 3*, 392-399. Berlin-Heidelberg-New York, Springer 1973.

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Adolf Hildebrand

Department of Mathematics
University of Illinois
1409 West Green Street
Urbana, Illinois 61801
U.S.A.