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## THE TRACE AS AN ALGEBRA HOMOMORPHISM

by Howard OSBORN

### 0. INTRODUCTION

Let  $A$  be any endomorphism of an appropriately restricted module  $V$  over a commutative ring with unit. The coefficients of the characteristic polynomial of  $A$  are the *elementary invariants* of  $A$ , being *traces* of  $A$ -induced endomorphisms of the exterior powers  $\wedge^p V$ . Similarly the *sums-of-powers invariants* of  $A$  are *traces* of the compositions  $A, AA, \dots$  of  $A$  with itself. For example, if  $V$  is free of rank  $n$  and  $A$  is represented by a diagonal matrix with diagonal entries  $t_1, \dots, t_n$ , then the elementary invariants and the sums-of-powers invariants are the usual elementary symmetric polynomials  $\sigma_1, \dots, \sigma_n$  and sums-of-powers polynomials  $s_1, s_2, \dots$ , respectively, in  $t_1, \dots, t_n$ . Since  $s_1, s_2, \dots$  can be expressed as the Newton polynomials in  $\sigma_1, \dots, \sigma_n$  one can easily use an appropriate “splitting principle” to prove that the sums-of-powers invariants of any endomorphism  $A$  of an appropriately restricted module  $V$  are the Newton polynomials in the elementary invariants of the same endomorphism  $A$ . The technique applies equally well to other trace-induced invariants of  $A$ .

In this intentionally elementary note such relations among the invariants of  $A$  are presented from a different point of view as images under the trace of identities in a new endomorphism algebra associated to the module  $V$ . Specifically, if  $\text{End } \wedge^p V$  denotes the module of endomorphisms of the  $p^{\text{th}}$  exterior power  $\wedge^p V$  of  $V$ , then one can provide the direct sum  $\coprod_p \text{End } \wedge^p V$  with a new product for which the trace becomes an *algebra homomorphism* onto the ground ring, preserving products as well as sums. There are universal identities in  $\coprod_p \text{End } \wedge^p V$  which express relations among the various endomorphisms induced by any endomorphism  $A$  of  $V$  itself, and one applies the trace to obtain corresponding identities among the invariants of  $A$  in the ground ring. The Newton identities are presented in this form to illustrate the technique.