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Autor: Osborn, Howard

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THE TRACE AS AN ALGEBRA HOMOMORPHISM

by Howard OSBORN

0. INTRODUCTION

Let A be any endomorphism of an appropriately restricted module V over a commutative ring with unit. The coefficients of the characteristic polynomial of A are the *elementary invariants* of A, being *traces* of A-induced endomorphisms of the exterior powers $\wedge^p V$. Similarly the *sums-of-powers invariants* of A are *traces* of the compositions $A, AA, \ldots$ of A with itself. For example, if V is free of rank n and A is represented by a diagonal matrix with diagonal entries $t_1, \ldots, t_n$, then the elementary invariants and the sums-of-powers invariants are the usual elementary symmetric polynomials $\sigma_1, \ldots, \sigma_n$ and sums-of-powers polynomials $s_1, s_2, \ldots$, respectively, in $t_1, \ldots, t_n$. Since $s_1, s_2, \ldots$ can be expressed as the Newton polynomials in $\sigma_1, \ldots, \sigma_n$ one can easily use an appropriate "splitting principle" to prove that the sums-of-powers invariants of any endomorphism A of an appropriately restricted module V are the Newton polynomials in the elementary invariants of the same endomorphism A. The technique applies equally well to other trace-induced invariants of A.

In this intentionally elementary note such relations among the invariants of A are presented from a different point of view as images under the trace of identities in a new endomorphism algebra associated to the module V. Specifically, if End $\wedge^p V$ denotes the module of endomorphisms of the p^{th} exterior power $\wedge^p V$ of V, then one can provide the direct sum $\amalg_p$ End $\wedge^p V$ with a new product for which the trace becomes an *algebra* homomorphism onto the ground ring, preserving products as well as sums. There are universal identities in $\amalg_p$ End $\wedge^p V$ which express relations among the various endomorphisms induced by any endomorphism A of V itself, and one applies the trace to obtain corresponding identities among the invariants of A in the ground ring. The Newton identities are presented in this form to illustrate the technique.

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