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XII. THE CLOSED GRAPH THEOREM

Let  $\mathfrak{E}, \mathfrak{F}$  be definite spaces in the sense of Definition 15 over a field  $k$  whose valuation topology satisfies the 1. axiom of countability. For  $f: \mathfrak{E} \rightarrow \mathfrak{F}$  a linear map set  $\mathfrak{G}(f) := \{(x, \eta) \in \mathfrak{E} \oplus \mathfrak{F} \mid \eta = f(x)\}$ . Then [21] the “closed graph theorem” can be proved by classical methods (Baire category arguments):

$$(24) \quad \mathfrak{G}(f) \text{ is closed} \Rightarrow f \text{ is continuous .}$$

There is the following algebraic analogue of statement (24):

$$(25) \quad \mathfrak{G}(f) = \mathfrak{G}(f)^{\perp\perp} \Rightarrow f \text{ is } \perp\text{-continuous}$$

Here  $\mathfrak{G}(f)^{\perp\perp}$  is taken in  $\mathfrak{E} \oplus^{\perp} \mathfrak{F}$  and, by definition,  $f$  is  $\perp$ -continuous iff  $f$  is continuous with respect to the topologies on  $\mathfrak{E}$  and  $\mathfrak{F}$  whose 0-neighbourhood filters are generated by the orthogonals of all *finite* dimensional subspaces of  $\mathfrak{E}$  and  $\mathfrak{F}$  respectively. For  $\mathfrak{E}$  an orthomodular space implication (25) holds:  $\mathfrak{G}(f) = \mathfrak{G}(f)^{\perp\perp}$  implies that  $\mathfrak{G}(f)$  is closed since the form is continuous on  $\mathfrak{E} \oplus^{\perp} \mathfrak{F}$ ; so  $f$  is continuous by (24). Further, if  $\mathfrak{G} \subset \mathfrak{F}$  is the orthogonal of a finite dimensional subspace then  $f^{-1}(\mathfrak{G})$  is closed, hence  $f^{-1}(\mathfrak{G}) = (f^{-1}(\mathfrak{G}))^{\perp\perp}$  as  $\mathfrak{E}$  is orthomodular. But  $(f^{-1}(\mathfrak{G}))^{\perp}$  is finite dimensional, hence  $f$  is  $\perp$ -continuous.

In [31] nice examples of  $f: \mathfrak{E} \rightarrow \mathfrak{F}$  are given which illustrate that (25) is in general violated.

XIII. A FEW OPEN PROBLEMS

All orthomodular spaces are meant to be infinite dimensional and different from the classical ones over  $\mathbf{R}, \mathbf{C}, \mathbf{H}$ .

*Problem 1.* Are cardinalities of maximal orthogonal families in an orthomodular space always equal? The answer is “yes” for those in  $\mathcal{E}$ .

*Problem 2.* Give an example of an orthomodular space that contains an uncountable orthogonal family of non-zero vectors.

*Problem 3.* Does the implication

$$\mathfrak{A} + \mathfrak{B} = (\mathfrak{A} + \mathfrak{B})^{\perp\perp} \Rightarrow \mathfrak{A}^{\perp} + \mathfrak{B}^{\perp} = (\mathfrak{A} \cap \mathfrak{B})^{\perp}$$