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THEOREM 34 [21]. Let  $\mathfrak{E}$  be a norm-topological space in the sense of Definition 31 and assume  $\varphi(2) = 0$ . Then the statements (i), (ii), (iii) in Theorem 28 are equivalent.

Remark 35. In Definition 15, Lemma 33 and in Theorem 34 we stipulated that  $\varphi(2) = 0$  for the valuation  $\varphi$  of the base field. However, it is neither necessary to assume this nor that  $\text{char } k$  be different from two. As technicalities increase if 2 is not a unit for  $\varphi$  the general case has been banned from this elementary survey. Refer to [21].

#### IX. APPENDIX: ORTHOMODULAR SPACES OVER ORDERED FIELDS

A Baer order of a  $*$ -field  $k$  is a subset  $\Pi \subset S := \{\alpha \in k \mid \alpha = \alpha^*\}$  with  $1 \in \Pi$ ,  $0 \notin \Pi$ ,  $\Pi + \Pi \subset \Pi$ ,  $\forall \alpha \neq 0: \alpha\Pi\alpha^* \subset \Pi$ ,  $-\Pi \cup \Pi = S \setminus \{0\}$ . ([14]). The map  $\alpha \mapsto \alpha^*\alpha =: \|\alpha\|$  has the properties of a norm and defines a topology on  $k$ ; if  $*$  is continuous then  $k$  is a topological  $*$ -field [14, Theorem 4.1, p. 231]. The theory of positive definite orthomodular spaces over archimedean ordered fields is settled in [9]: There are but the classical Hilbert spaces over  $\mathbf{R}$ ,  $\mathbf{C}$ ,  $\mathbf{H}$ . If the order is non-archimedean we shall assume that

(15) the subgroup  $S$  generated by all  $\alpha^*\alpha^{-1}$  is bounded.

There is [14, Sec. 4.5, p. 234] a valuation on  $k$  that induces the norm-topology. We remark that the boundness condition on  $S$  is always satisfied for the usual orderings on commutative fields, for Prestel's semi-orderings and for all  $*$ -ordered fields that are known hitherto.

A family  $(e_\nu)_{\nu \in I}$  of vectors in a positive definite space  $(\mathfrak{E}; \langle \cdot, \cdot \rangle)$  over an ordered  $*$ -field  $k$  is said to satisfy the type condition (cf. Definition 21) iff for all  $(\alpha_\nu)_{\nu \in I} \in k^I$  the following holds: if  $(\langle \alpha_\nu e_\nu \rangle)_{\nu \in I}$  is bounded then  $(\alpha_\nu e_\nu)_{\nu \in I}$  converges to  $0 \in \mathfrak{E}$ .

With this version of type condition we have

THEOREM 36. Let  $(\mathfrak{E}; \langle \cdot, \cdot \rangle)$  be a positive definite space over a non-archimedean ordered  $*$ -field that satisfies (15). Then the statements (i), (ii), (iii) in Theorem 28 are equivalent.