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THEOREM 34 [21]. Let \mathfrak{E} be a norm-topological space in the sense of Definition 31 and assume $\varphi(2) = 0$. Then the statements (i), (ii), (iii) in Theorem 28 are equivalent.

Remark 35. In Definition 15, Lemma 33 and in Theorem 34 we stipulated that $\varphi(2) = 0$ for the valuation φ of the base field. However, it is neither necessary to assume this nor that $\text{char } k$ be different from two. As technicalities increase if 2 is not a unit for φ the general case has been banned from this elementary survey. Refer to [21].

IX. APPENDIX: ORTHOMODULAR SPACES OVER ORDERED FIELDS

A Baer order of a $*$ -field k is a subset $\Pi \subset S := \{\alpha \in k \mid \alpha = \alpha^*\}$ with $1 \in \Pi$, $0 \notin \Pi$, $\Pi + \Pi \subset \Pi$, $\forall \alpha \neq 0: \alpha\Pi\alpha^* \subset \Pi$, $-\Pi \cup \Pi = S \setminus \{0\}$. ([14]). The map $\alpha \mapsto \alpha^*\alpha =: \|\alpha\|$ has the properties of a norm and defines a topology on k ; if $*$ is continuous then k is a topological $*$ -field [14, Theorem 4.1, p. 231]. The theory of positive definite orthomodular spaces over archimedean ordered fields is settled in [9]: There are but the classical Hilbert spaces over \mathbf{R} , \mathbf{C} , \mathbf{H} . If the order is non-archimedean we shall assume that

(15) the subgroup S generated by all $\alpha^*\alpha^{-1}$ is bounded.

There is [14, Sec. 4.5, p. 234] a valuation on k that induces the norm-topology. We remark that the boundness condition on S is always satisfied for the usual orderings on commutative fields, for Prestel's semi-orderings and for all $*$ -ordered fields that are known hitherto.

A family $(e_l)_{l \in I}$ of vectors in a positive definite space $(\mathfrak{E}; \langle \cdot, \cdot \rangle)$ over an ordered $*$ -field k is said to satisfy the type condition (cf. Definition 21) iff for all $(\alpha_l)_{l \in I} \in k^I$ the following holds: if $(\langle \alpha_l e_l \rangle)_{l \in I}$ is bounded then $(\alpha_l e_l)_{l \in I}$ converges to $0 \in \mathfrak{E}$.

With this version of type condition we have

THEOREM 36. Let $(\mathfrak{E}; \langle \cdot, \cdot \rangle)$ be a positive definite space over a non-archimedean ordered $*$ -field that satisfies (15). Then the statements (i), (ii), (iii) in Theorem 28 are equivalent.