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## A HOLOMORPHICALLY SEPARABLE COMPLEX SPACE WITHOUT THE GELFAND TOPOLOGY

by Sandra HAYES-WIDMANN

### ABSTRACT

An example of a holomorphically separable complex space with a Stein envelope of holomorphy which does not carry the Gelfand topology is given. This example also shows that an injective holomorphic map  $\varphi: X \rightarrow Y$  between complex spaces with  $\dim_x X = \dim_{\varphi(x)} Y$ ,  $x \in X$ , is not always open, even when  $\varphi$  is the canonical map of a pre-Stein space  $X$  into its envelope of holomorphy.

### INTRODUCTION

The Gelfand topology for a reduced complex space  $(X, \mathcal{O})$  is the weak topology on  $X$  determined by the global function algebra  $\mathcal{O}(X)$ . Since only holomorphically separable complex spaces can carry this topology, it is natural to ask whether holomorphic separability characterizes those complex spaces with the Gelfand topology. A remark in [4, Bemerkung 3] implies that this is the case, at least for pre-Stein spaces. However, a counter-example given here shows that holomorphically separable spaces need not have the Gelfand topology, even when they are pre-Stein.

### EXAMPLE

If a complex space  $(X, \mathcal{O})$  is furnished with the Gelfand topology, then it must be holomorphically separable in a strong sense—every interior point can be separated not only from every other interior point but also from every “boundary” point by a global holomorphic function. More precisely,

the latter separation property means that for every point  $x \in X$  and for every sequence  $(x_n)_{n \in \mathbf{N}}$  in  $X \setminus \{x\}$  with no cluster point in  $X$ , there exists a global holomorphic function  $f \in \mathcal{O}(X)$  such that

$$f(x) \notin \overline{\{f(x_n) \mid n \in \mathbf{N}\}}.$$

The following example shows that holomorphically separable complex spaces having interior points which cannot be separated from boundary points actually exist. I am indebted to J. P. Vigué for the construction involved in this example.

In  $\mathbf{C}^3$  with the coordinates  $x, y, z$  denote by

$$C := \left\{ (0, y, 0) \in \mathbf{C}^3 \mid |y| \leq \frac{1}{2} \right\}$$

the circle with radius  $1/2$  around the origin in the  $y$ -plane. In  $\{0\} \times \mathbf{C}^2$  let

$$Y := \left\{ (0, y, z) \in \mathbf{C}^3 \mid |y| < 1, |z| < 1 \right\} \setminus C$$

be the unit bidisc with  $C$  omitted. Let  $Z$  be the unit bidisc in  $\mathbf{C}^2 \times \{0\}$  with the circumference of  $C$  elected, i.e.

$$Z := \left\{ (x, y, 0) \in \mathbf{C}^3 \mid |x| < 1, |y| < 1 \right\} \setminus \left\{ (0, y, 0) \in \mathbf{C}^3 \mid |y| = \frac{1}{2} \right\}.$$

The ring

$$R := \left\{ (0, y, 0) \in \mathbf{C}^3 \mid \frac{1}{2} < |y| < 1 \right\}$$

is an analytic subset of  $X$  as well as of  $Y$ . Attach  $Y$  to  $Z$  along  $R$  and call the resulting space  $X$ . This space, which is the fiber sum (pushout)  $Y +_R Z$  of  $Y$  and  $Z$  under the inclusions  $R \rightarrow Y$  and  $R \rightarrow Z$ , is a holomorphically separable complex space [2].

$X$  cannot have the Gelfand topology. To see this, observe that  $X$  is the disjoint union of  $Y$  and  $Z$  with points of  $R$  identified. Consequently, the origin in  $\mathbf{C}^3$  can be considered as an interior point in  $X$ , since it is a point in  $Z$ , as well as a point not belonging to  $X$ , since it isn't in  $Y$ . These two points can't be separated by any global holomorphic function.

$X$  is a pre-Stein space, i.e.  $X$  has a Stein envelope of holomorphy as defined in [1]. This follows from the fact that  $\mathcal{O}(X)$  is the algebra  $\mathcal{O}(D)$  of global holomorphic functions on the Stein space  $D := D_1 +_E D_2$  obtained by attaching the unit bidisc  $D_1$  in  $\{0\} \times \mathbb{C}^2$  to the unit bidisc  $D_2$  in  $\mathbb{C}_2 \times \{0\}$  along

$$E := \left\{ (0, y, 0) \in \mathbb{C}^3 \mid |y| < 1 \right\}.$$

It is well known that every Stein space is equipped with the Gelfand topology (see [1]).

There is a classical dimension formula [3, Sätze 28, 29] for an injective holomorphic map  $\varphi: X \rightarrow Y$  between complex spaces where  $Y$  is locally irreducible which states that  $\varphi$  is open, if  $\dim_x X = \dim_{\varphi(x)} Y$  for  $x \in X$ . According to the above example, this formula cannot be generalized to maps  $\varphi: X \rightarrow Y$  if  $Y$  is not locally irreducible, not even when  $Y$  is the Stein envelope of holomorphy of  $X$  and  $\varphi$  is the canonical map which takes points  $x$  of  $X$  to the corresponding point evaluations  $\mathcal{O}(X) \mapsto \mathbb{C}, f \rightarrow f(x)$ , in the continuous spectrum  $S_c(\mathcal{O}(X))$ .

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