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5.5. *Remark.* From the preceding proof we can derive the upper bound $\exp \exp 4n^2$ for the number of isomorphism classes of n-dimensional crystallographic groups. The correct numbers for $n = 1, 2, 3, 4$ are respectively 2, 17, 219, 4783 [4].

REFERENCES

The original articles of Bieberbach and Frobenius are

[1] BIEBERBACH, L. Über die Bewegungsgruppen der Euklidischen Räume, I: *Math. Ann. 70* (1911), 297-336; II: *Math. Ann. 72* (1912), 400-412.

[2] FROBENIUS, C. Über die unzerlegbaren diskreten Bewegungsgruppen. *Sitzungsber. Akad. Wiss. Berlin 29* (1911), 654-665.

A simplified version of Frobenius'proof using minor amounts of Lie group theory is in

[3] AUSLANDER, L. An account of the theory of crystallographic groups. *Proc. Amer. Math. Soc. 16* (1956), 1230-1236.

For historical remarks we refer to

[4] BROWN, H., R. BÜLOW, J. NEUBÜSER, H. WONDRATSCHEK and H. ZASSENHAUS. *Crystallographic Groups of Four Dimensional Space.* New York, Wiley, 1978.

[5] BUSER, P. and H. KARCHER. The Bieberbach Case in Gromov's Almost Flat Manifold Theorem. In *Global Differential Geometry and Global Analysis, Proceedings, Berlin 1979, Lect. Notes in Mathematics, 838*, Berlin, Springer Verlag, 1981.

[6] MILNOR, J. Hilbert's Problem 18: On Crystallographic Groups, Fundamental Domains, and on Sphere Packing. *Proceedings of Symposia in Pure Mathematics, 28*, Amer. Math. Soc., Providence, 1976.

A proof of Bieberbach's third theorem that two crystallographic groups are isomorphic if and only if they are conjugate by an affine transformation, may be found on p. 375 in

[7] RINOW, W. *Die innere Geometrie der metrischen Räume.* Berlin, Springer Verlag, 1961.

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