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A NOTE ON LEVI'S PROBLEM WITH DISCONTINUOUS FUNCTIONS

by Mihnea COLTOIU

§ 1. INTRODUCTION

In [3] Fornaess and Narasimhan proved that a complex space X which carries a strongly plurisubharmonic exhaustion function $\varphi: X \rightarrow \mathbf{R}$ is a Stein space. It is a remarkable fact that φ is supposed only upper semicontinuous.

A natural question which arises when we consider the Levi problem with upper semicontinuous functions is the following: what would happen if we allowed φ to take on the value $-\infty$. Simple examples (compact complex spaces, the blowing up of \mathbf{C}^n at the origine...) show us that X is not necessarily Stein. The best result one might hope to obtain is X being 1-convex.

The aim of this short note is to give an affirmative answer to this question, hence to prove the following theorem conjectured by Fornaess and Narasimhan:

THEOREM 1. *Let X be a complex space which admits a strongly plurisubharmonic exhaustion function $\varphi: X \rightarrow [-\infty, \infty)$. Then X is 1-convex.*

If φ is supposed real-valued it follows easily, from the maximum principle, that the exceptional set of X is empty, hence X is Stein. This is exactly Fornaess-Narasimhan's theorem.

§ 2. PRELIMINARIES

All complex spaces are assumed to be reduced and countable at infinity.

An upper semicontinuous function $\varphi: X \rightarrow [-\infty, \infty)$ is called plurisubharmonic if for every holomorphic map $\tau: W \rightarrow X$ ($W =$ the unit disc in \mathbf{C}) it follows that $\varphi \circ \tau$ is subharmonic on W (possibly $\equiv -\infty$). φ is said

to be strongly plurisubharmonic if for every C^∞ real-valued function θ with compact support there exists an $\varepsilon_0 > 0$ such that $\varphi + \varepsilon\theta$ is plurisubharmonic for $|\varepsilon| \leq \varepsilon_0$.

A main result in [3] tells us that the above definition agrees with the usual one as given in [6].

Let us also recall that a complex space X is said to be 1-convex if there exist:

- i) a compact analytic set $S \subset X$ with $\dim_x S > 0$ for any $x \in S$,
- ii) a Stein space Y , a finite set $A \subset Y$ and a proper holomorphic map $p: X \rightarrow Y$ inducing a biholomorphism $X \setminus S \cong Y \setminus A$ and which satisfies $p_* \mathcal{O}_X \cong \mathcal{O}_Y$.

S is called the exceptional set of X and Y the Remmert reduction of X .

Remark. Using the analytic version of Chow's lemma (Hironaka [5]) it was proved in [2] that any 1-convex space X carries a strongly plurisubharmonic exhaustion function $\varphi: X \rightarrow [-\infty, \infty)$, i.e. the converse of Theorem 1 holds too.

§ 3. THE PROOF OF THEOREM

We shall apply Andreotti-Grauert's technique [1] with suitable modifications required by the upper semicontinuity. Throughout this section \mathcal{F} will denote a coherent sheaf on X and $X_c = \{x \in X \mid \varphi(x) < c\}$.

To prove Theorem 1 we need some lemmas.

LEMMA 1. *For any $c \in \mathbf{R}$ there exists $\varepsilon > 0$ such that restriction map $H^1(X_{c+\varepsilon}, \mathcal{F}) \rightarrow H^1(X_{c+\varepsilon'}, \mathcal{F})$ is surjective for any $0 \leq \varepsilon' \leq \varepsilon$.*

Proof. We may assume $c = 0$. Set $K = \overline{\{\varphi < 1\}}$ and let $\{U_1, \dots, U_m\}$ be a covering of K with Stein open sets, $U_i \subset \subset X$ and $h_i \in C_0^\infty(U_i)$, $h_i \geq 0$ such that $\varphi - \sum_{i=1}^r h_i$ is strongly plurisubharmonic for $r = 1, \dots, m$ and $\sum_{i=1}^m h_i > 0$ on K . Choose $\alpha > 0$ such that $\sum_{i=1}^m h_i(x) \geq \alpha$ for any $x \in K$ and take $0 < \varepsilon < \min(\alpha, 1)$. We shall prove that this ε satisfies the conditions required in Lemma 1.

For any $0 \leq \varepsilon' \leq \varepsilon$ we set $X_{\varepsilon'}^r = \{x \in X \mid \varphi(x) < \varepsilon' + h_1(x) + \dots + h_r(x)\}$ for $r = 0, \dots, m$ (by definition $X_{\varepsilon'}^0 = X_{\varepsilon'}$).